Practice Exam for second Midterm

March 28, 2013

1. Convert to polar coordinates the following Cartesian coordinates
   (a) $(-1, 0)$,
   (b) $(\sqrt{6}, -\sqrt{2})$,
   (c) $(-\sqrt{3}, 1)$.
2. Find the area inside the spiral $r = \theta$ for $0 \leq \theta \leq 2\pi$.
3. Find the arclength of the cartioid $r = 1 - \sin\theta$ between $\theta = 0$ and $\theta = \pi/2$.
4. Find the sum of the finite series $(0.5)^3 + (0.5^4 + (0.5)^5 + \ldots + (0.5)^{2013}$.
5. Calculate the sum $1 + 2 + 5 + 6 + 9 + 10 + 13 + 14 + \ldots + 2009 + 2010 + 2013 + 2014$.
6. Calculate the sum of the infinite series $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \ldots$.
7. Determine whether the following sequences converge or diverge:
   (a) $\frac{3 + 4n}{5 + 7n}$
   (b) $\frac{1}{n} + \ln n$
   (c) $\sin\left(\frac{\pi}{4}n\right)$.
8. Determine which of the series converge:
   (a) $\sum_{n=1}^{\infty} \frac{1}{(n+2)^2}$
   (b) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

(d) $\sum_{n=1}^{\infty} \frac{n^3 - 2n^2 + n + 1}{n^5 - 2}$

(e) $\sum_{n=1}^{\infty} 2^{-n} \frac{(n+1)}{(n+2)}$

9. A new car costs \$30,000; it loses 10% of its value each year. Maintenance is \$500 the first year and increases by 20% annually.

   (a) Find a formula for l_n, the value lost by the car in year n.

   (b) Find a formula for m_n, the maintenance expenses in year n.

   (c) In what year do the maintenance expenses first exceed the value lost by the car?

10. True/False

   (a) You can tell if a sequence converges by looking at the first 1000 terms.

   (b) If the sequence s_n of positive terms is unbounded, then the sequence has a term greater than a million.

   (c) If all terms s_n of a sequence are less than a million, then the sequence is bounded.

   (d) The series $\sum_{n=1}^{\infty} 2^{(-1)^n}$ converges.

   (e) If $\sum a_n b_n$ converges then $\sum a_n$ and $\sum b_n$ converge.

   (f) If $a_n > 0.5 b_n > 0$ for all n and $\sum b_n$ diverges, then $\sum a_n$ diverges.

   (g) If $\int_0^{\infty} f(x)\,dx$ converges, then $\int_0^{\infty} 7f(x)\,dx$ converges.

   (h) If $\int_0^{\infty} f(x)\,dx$ converges, then $\int_0^{\infty} f(7x)\,dx$ converges.