Math 53
Homework 3

due February 13 in class

For next week, you have to pick one of these to present in class. The proofs of all of them are in pages 96 and 97 of the textbook. All six exercises combine to prove Theorem 3.14 in the textbook. You need not write up any of the solutions to turn in, you're only homework is to present in class.

1. Prove
$$\sum_{d\le x} \frac{\Lambda(d)}{d} = \log x + O(1).$$

2. Prove
$$\sum_{\substack{p^k\le x\\ k\ge 2}} \frac{\log p}{p^k} = O(1).$$

3. Using (1) and (2) show that
$$\sum_{p\le x} \frac{\log p}{p} = \log x + O(1).$$

4. Let $A(x) = \sum_{p\le x} \frac{\log p}{p}$. Show
$$\sum_{p\le x} \frac{1}{p} = \frac{A(x)}{\log x} + \int_2^x \frac{A(t)}{t\log^2 t}dt \qquad \text{and}$$
$$\int_2^x \frac{A(t)-\log t}{t\log^2 t}dt = O(1).$$

5. Show
$$\int_x^\infty \frac{A(t)-\log t}{t\log^2 t}dt = O\left(\frac{1}{\log x}\right).$$

6. Using (4) and (5) prove
$$\sum_{p\le x} \frac{1}{p} = \log\log x + B_1 + O\left(\frac{1}{\log x}\right),$$
where $B_1 = 1 - \log\log 2 + \int_2^\infty \frac{A(t)-\log t}{t\log^2 t}dt.$