

Math 53
Homework 5

due February 27 in class

Let

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1, \\ (-1)^k & \text{if } n = p_1 p_2 \dots p_k \text{ for distinct primes } p_1, \dots, p_k, \\ 0 & \text{if } n \text{ has a square divisor greater than 1.} \end{cases}$$

For example $\mu(7) = (-1)^1 = -1$, $\mu(6) = (-1)^2 = 1$, $\mu(12) = 0$.

1. Prove that μ is a multiplicative function, i.e., that $\mu(mn) = \mu(m)\mu(n)$ for any m and n with $\gcd(m, n) = 1$.
2. Prove that

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{if } n > 1 \text{ is an integer.} \end{cases}$$

3. Exercise 20 in Chapter 3.
4. Exercise 21 in Chapter 3.