Math 53
Homework 8

due April 3 in class

1. Show that χ is a primitive character modulo q if and only if for all $d \mid q$ there exists an $a \equiv 1 \bmod d$ with $(a, q) = 1$ such that $\chi(a) \neq 1$.

2. Show that there are no primitive characters modulo q for $q \equiv 2 \bmod 4$.

3. Let $q > 1$ be an integer and let $\zeta = e^{2\pi i/q}$ be a primitive q-th root of unity. Let χ be a Dirichlet character modulo q. Let

$$\tau(\chi) = \sum_{n=1}^{q} \chi(n)\zeta^n,$$

and for a an integer,

$$\tau_a(\chi) = \sum_{n=1}^{q} \chi(n)\zeta^{an}.$$

In class we showed that if $(a, q) = 1$, then $\tau_a(\chi) = \bar{\chi}(a)\tau(\chi)$. Show that if χ is a primitive character, then $\tau_a(\chi) = 0$ for $(a, q) > 1$, which would show that for χ primitive, $\tau_a(\chi) = \bar{\chi}(a)\tau(\chi)$ for all integers a and hence make our proof of Pólya–Vinogradov complete.

4. Show that

$$\sum_{n=1}^{\infty} \frac{\cos n}{\sqrt{n}}$$

converges.