# Math 53
# Homework 9

due April 17 in class

1. For the following two complex numbers find the real part, the imaginary part, the modulus and the argument:
   (a) $\dfrac{1+3i}{7+2i}$.
   (b) $\dfrac{1+\sqrt{-3}}{2}$.
2. Using the Cauchy-Riemann equations show that $f(z) = z^3$ is analytic for all $z \in \mathbb{C}$.
3. Let $f(z) = \dfrac{1}{\sin z}$.
   (a) Find the first three nonzero terms of the Laurent series of $f(z)$ about $z = 0$.
   (b) What is the residue of $f(z)$ at $z = 0$.
4. Let $N$ be a positive integer. Let $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ be smooth paths from $[-(N+1/2), N+1/2]$ to $\mathbb{C}$ be defined as

$$\begin{aligned}
\gamma_1(t) &= (N+1/2) + i(t)\\
\gamma_2(t) &= -t + i(N+1/2)\\
\gamma_3(t) &= -(N+1/2) + i(-t)\\
\gamma_4(t) &= t + i(-(N+1/2)).
\end{aligned}$$

   Let $\gamma_N$ be the closed path created by concatenating $\gamma_1, \gamma_2, \gamma_3, \gamma_4$.
   (a) Evaluate $\displaystyle\int_{\gamma_1} z^2\, dz$.
   (b) Evaluate $\displaystyle\int_{\gamma_N} z^2\, dz$.
   (c) Evaluate $\displaystyle\int_{\gamma_N} \frac{1}{z^2 - N^2}\, dz$.
5. Let $\gamma$ be the path centered at $-N$ with radius $N$ (going in a clockwise direction).
   (a) Using the residue theorem, evaluate $\displaystyle\int_{\gamma} \frac{1}{z^2 - N^2}\, dz$.
   (b) Show that the path $\gamma$ can be written as $\gamma(t) = -N(e^{it} + e^{-it})$ for $t \in [0, 2\pi]$.
   (c) Evaluate $\displaystyle\int_{\gamma} \frac{1}{z^2 - N^2}\, dz$ the long way using (b).