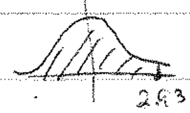
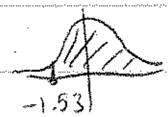
Practice Exam 3
Solutions

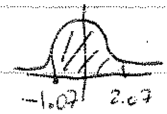
(1)

a) 

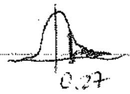
Using table A-2 we find that the probability is $\boxed{0.9983}$

b) 

The area on the left is 0.0630. Therefore the prob. is $\boxed{0.9370}$ $(1-.0630)$

c) 

The prob is $0.9808-0.1423=\boxed{0.8385}$

d) 

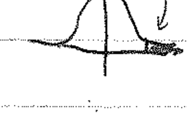
$\mu=0$, $\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{16}}=\frac{1}{4}$.

so $z_{\bar{x}}=\frac{0.27-0}{\left(\frac{1}{4}\right)}=1.08$.

The area on the left is 0.8599, so the prob is $1-0.8599=\boxed{0.1401}$

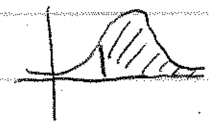
(2) $\mu=21.1$, $\sigma=5.1$, $n=80$

a) The distribution is normal.
b) The mean of the sample means is $\mu=21.1$.
c) The standard deviation of the sample means is
$$\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}=\frac{5.1}{\sqrt{80}}\approx 0.5702$$

d) $z_{\bar{x}}=\frac{23-21.1}{\sigma_{\bar{x}}}=\frac{1.9}{0.5702}\approx 3.33.$ 
area on the right.

The area on the left is 0.9996
So the probability is $1-0.9996=\boxed{0.0004}$

(3) $\mu = 1634\text{mm}$, $\sigma = 66\text{mm}$

a) 

$$z = \frac{1500-1634}{66} = \frac{-134}{66} \approx -2.03$$

The area on the left is 0.0212
Therefore the probability is $1 - .0217 = \boxed{0.9788}$

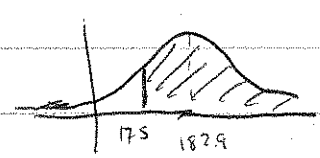
b) We want to find a z-score z such that the area to the left is 0.9500. The z-score is 1.645.
So $z = 1.645$.
It asks for the height so we translate it to a height:

$$1.645 = z = \frac{x-\mu}{\sigma} = \frac{x-1634}{66}.$$

So $x = (1.645)(66) + 1634 = 1742.57$
The answer is $\boxed{1742.57\text{mm}}$ (you could round it).

NOTE: The question is ill-posed because the 95th percentile is comfortable for the tallest 5% not for the lowest 95%.

4) a) Since the male has a carry-on you can assume $x = 195 - 20 = 175$.



$$z = \frac{175 - 182.9}{40.9} = -0.19315 \doteq -0.19$$

So the area on the left is 0.4247
So the probability is $1 - 0.4247 = \boxed{0.5753}$

b) Because of the carry on we want the sample mean to be greater than 175 instead of 195.

$\mu = 182.9$
$\sigma = 40.9$ so $\sigma_{\bar{x}} = \dfrac{40.9}{\sqrt{213}} = 2.8024$

Therefore $z_{\bar{x}} = \frac{175 - 182.9}{2.8024} \approx -2.82$.

The area to the left of -2.82 is 0.0024.

So the probability is $\boxed{0.9976}$

The probability is very high, so the pilot should be concerned.

(5) a) $\hat{p} = \frac{284}{557} = 0.50987\ldots \approx 0.51$

b) 95% confidence interval.

Therefore $z_{\alpha/2} = 1.96$

So

$$E = 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(0.51)(0.49)}{557}} = 0.0415\ldots$$

$E \approx 0.042$

$$\hat{p} - E < p < \hat{p} + E$$

Therefore $0.468 < p < 0.552$

c) You can't since the confidence interval allows values below 0.5.

(6) We use the formula $n = \left(\frac{z_{\alpha/2}}{E}\right)^2\left(\frac{1}{4}\right)$ because $\hat{p}$ is unknown.

Since the confidence level is 0.99, $z_{\alpha/2} = 2.575$

so $n = \left(\frac{2.575}{.02}\right)^2\left(\frac{1}{4}\right) = 4144.14\ldots$

we round up to get $\boxed{n = 4145}$

(7) a) $\boxed{\text{Student } t}$, because σ is unknown and $n > 30$.

b) $\boxed{\text{Normal}}$, because σ is known and $n > 30$.

c) $\boxed{\text{None}}$, because the population is skewed and to estimate the standard deviation we require it to be normal.

d) $\boxed{\chi^2}$, because we're estimating σ and the population has a normal distribution.

e) $\boxed{\text{Normal}}$, because $n\hat{p} \geq 5$ $((850)(0.1) = 85)$
and $n\hat{q} \geq 5$ $((850)(0.9) = 765)$.

(8) a) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \left(\frac{1}{4}\right)$. Since $1-\alpha = 0.98$ then $\alpha = .02$ so $\alpha/2 = .01$ so $1-\alpha/2 = .99$

Therefore $\boxed{z_{\alpha/2} = 2.33}$ (using Table A-2).

So $n = \left(\frac{2.33}{.05}\right)^2 \left(\frac{1}{4}\right) = 542.89$.

Therefore $\boxed{n = 543}$

b) Since we're estimating the mean: $n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2$

$\sigma = 337$, $E = 50$, $z_{\alpha/2} = 2.33$

so

$$n = \left(\frac{(2.33)(337)}{50}\right)^2 = 246.62$$

so

$\boxed{n = 247}$

c) Since 543 is the largest of 543 and 247 we need a sample of at least 543.

$\boxed{n = 543}$

(9) a) $\bar{x} = 42.7g$
$s = 5.6g$
$n = 7$

$\boxed{\text{Since } \sigma \text{ is unknown, we use the t-test.}}$

$1-\alpha = .95$ so $\alpha = .05$

Therefore $t_{\alpha/2} = 2.447$ (using Table A-3)

(note that there are 6 degrees of freedom and the area in two tails is .05)

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}} = 2.447 \frac{5.6}{\sqrt{7}} = 5.179\ldots$$
$$\approx 5.18.$$

So the CI for the mean is

$$\bar{x} - E < \mu < \bar{x} + E$$

$$\boxed{37.52 < \mu < 47.88}$$

b) Since we're estimating σ we use the χ^2 test.

$1-\alpha = .95$ so $\alpha = .05$ so $\frac{\alpha}{2} = .025$
and $1-\frac{\alpha}{2} = .975$

For χ^2_R we use Table A4 with 6 degrees of freedom and area to the right $= 0.025$

so

$$\chi^2_R = 14.449$$

For χ^2_L we use Table A4 with 6 degrees of freedom and area to the right 0.975

so $\chi^2_L = 1.237$

So the CI for σ is

$$\left(\sqrt{\frac{n-1}{\chi_R^2}}\right) s < \sigma < \left(\sqrt{\frac{n-1}{\chi_L^2}}\right) s$$

$$\left(\sqrt{\frac{6}{14.449}}\right)(5.6) < \sigma < \left(\sqrt{\frac{6}{1.237}}\right)(5.6)$$

$$\boxed{3.61 < \sigma < 12.33}$$