Name: ____________________

Math 230 Midterm #1

February 7, 2014

Instructions: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- You may NOT use a calculator.
- Show all of your work.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	10	
7	20	
8	20	
Total:	150	

1. True or False (Just answer true or false, you don't need to explain your answer).
   (a) [2 points] $T \subseteq A$ if and only if $T \in 2^A$.

   (b) [2 points] There is no x such that $x \subseteq \{x\}$.

   (c) [2 points] If x is a real number and $x^2 < 0$, then x is a perfect number.

   (d) [2 points] Two right triangles that have hypotenuses of the same length have the same area.

   (e) [2 points] $\exists x, \forall y, xy = 0$.

   (f) [2 points] $\forall x, \exists y, xy = 0$.

   (g) [2 points] $\mathbb{N} \in 2^{\mathbb{Z}}$.

   (h) [2 points] $\{2\} \subseteq \{\{1\}, \{2\}, \{3\}\}$.

   (i) [2 points] If A and B are sets then $2^A \subseteq 2^B$.

   (j) [2 points] A negation of the statement "There is a natural number that is prime and even" can be phrased as "All natural numbers that are prime are odd".

2. For the following pairs of statements A, B, write a if the statement "If A, then B" is true, write b if the statement "If B, then A" is true, write c if the statement "A if and only if B is true", and write d if none of the statements are true. You should write all that apply. Note that in the following, x and y are integers.

   (a) [5 points] A: $xy = 0$. B: $x = 0$ and $y = 0$.

   (b) [5 points] A: Lines l_1 and l_2 are parallel. B: Lines l_1 and l_2 are perpendicular.

   (c) [5 points] A: Joe is a grandfather. B: Joe is male.

   (d) [5 points] A: $x < 0$ B: $x^3 < 0$.

3. Proofs:

   (a) [10 points] Let x be an integer. Prove that x is odd if and only if there is an integer b such that $x = 2b - 1$.

(b) [5 points] For real numbers a and b, prove that if $0 < a < b$, then $a^2 < b^2$

(c) [5 points] Let A, B and C be sets satisfying $A \subseteq B$ and $B \subseteq C$. Prove that $A \subseteq C$.

4. Find counterexamples to disprove the following statements:

   (a) [5 points] If a, b and c are positive integers with $a|(bc)$, then $a|b$ or $a|c$.

   (b) [5 points] Two right triangles have the same area if and only if the lengths of their hypotenuses are the same.

   (c) [5 points] For real numbers a and b, if $a < b$, then $a^2 < b^2$.

   (d) [5 points] Let A and B be sets. Then $(A \cup B) - B = A$.

5. Boolean Algebra

   (a) [5 points] Prove or disprove the following Boolean expression identity:

$$(x \wedge y) \vee (x \wedge \neg y) = x.$$

   (b) [5 points] Besides the classic Boolean operations $\wedge, \vee, \neg, \rightarrow, \leftarrow$, we have others, an example of one is the "nand" operation denoted by $\bar{\wedge}$. We define $x \bar{\wedge} y$ to be $\neg(x \wedge y)$. Construct a truth table for $\bar{\wedge}$.

(c) [5 points] Prove or disprove that $\bar{\wedge}$ is commutative.

(d) [5 points] Prove or disprove that $\bar{\wedge}$ is associative.

6. In my comic book library I have 15 Daredevil paperbacks, 12 Spider-man paperbacks and 3 Batman paperbacks

   (a) [5 points] In how many different ways can these trade paperbacks be arranged on a bookshelf?

   (b) [5 points] In how many different ways can these trade paperbacks be arranged on a bookshelf if all the books of the same character are grouped together?

7. Write out the following sets by listing their elements between curly braces.

   (a) [5 points] $\{x \in \mathbb{N} : x \leq 10 \text{ and } 3|x\}$.

   (b) [5 points] $\{x \in \mathbb{Z} : x^2 = 4\}$.

   (c) [5 points] $\{x \in \mathbb{Z} : 10|x \text{ and } x|100\}$.

   (d) [5 points] $\{x : x \subseteq \{1, 2, 3, 4, 5\} \text{ and } |x| \leq 1\}$.

8. Let $A \times B = \{(1,2),(1,3),(1,7),(2,2),(2,3),(2,7),(6,2),(6,3),(6,7)\}$.

   (a) [5 points] What is $A \cup B$?

   (b) [5 points] What is $A \cap B$?

   (c) [5 points] What is $A - B$?

   (d) [5 points] What is $A \Delta B$?