Extra Problem Homework 6 (π-day special)

Let $y(t)$ be the following function of t:

$$y(t) = \int_0^\infty e^{-x^2} \cos(2xt)\, dx.$$

(a) Prove that $y(t)$ satisfies the differential equation

$$y' + 2ty = 0.$$

(Hint: You have to compute y', then integrate it by parts).

(b) Find the general solution to this differential equation.

(c) In class we proved that

$$y(0) = \int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}.$$

Use this initial condition to show that

$$y(t) = \frac{\sqrt{\pi}}{2} e^{-t^2}.$$