# Practice Exam 1, Math 214

1. A tank contains 100 liters of water and 50 grams of a chemical. Water containing a concentration of $\frac{1}{4}\left(1+\frac{t}{2}\right)$ g/l of this chemical flows into the tank at a rate of 2 liters per minute, and the mixture flows out at the same rate.

   (a) Write a differential equation for the amount of chemicals in the tank at any time.

   (b) Find the amount of chemical in the tank at any time.

2. Find the solution of the initial value problem.

$$y' + 2y = te^{-2t}, \qquad y(1) = 0.$$

3. Find the general solution to:

$$\frac{dy}{dx} = \frac{x^2}{y}.$$

4. Without finding a solution, determine an interval in which the solution of the initial value problem is guaranteed to exist.

$$(4 - t^2)y' + 2ty = 3t^2, \qquad y(1) = -3.$$

5. Suppose a population $y$ is modeled by the equation

$$y' = -y\left(1 - \frac{y}{a}\right)\left(1 - \frac{y}{1000}\right).$$

   (a) For $a = 200$, sketch:
   - The graph of $y'$ as a function of $y$.
   - The phase line.
   - Several possible solution curves $y(t)$ including any equilibrium solutions.

   (b) For arbitrary $a > 0$, characterise the stability of the equilibrium solutions. Do not assume $a < 1000$.

   (c) Sketch a bifurcation diagram for the parameter $a$.

6. Find the general solution to the following differential equations. You do not have to justify that your solution is the general solution.

   (a) $y'' - 6y' + 18y = 0.$

   (b) $4y'' - 4y' + 3y = 0.$

   (c) $y'' - 6y' + 18y = 3e^{3t}.$

   (d) $y'' + y = \tan t.$

7. (a) Find two constants $n$ such that $y = t^n$ is a solution to the differential equation

$$t^2y'' + 3ty' - 3y = 0.$$

   (b) Write down the general solution to the differential equation for $t < 0$ and use the Wronskian to justify that this is the general solution.

8. Solve the initial value problem

$$y'' - 2y' + y = 3te^{2t}, \quad y(0) = 2 \;\; y'(0) = 4.$$