Homework 4 Solutions
Math 150

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3.2:

(a) The table gives $P(Z \leq -1.13) = 0.1292$. Therefore $P(Z > -1.13) = 1 - 0.1292 = 0.8708$.

(b) The table yields $P(Z \leq 0.18) = 0.5714$.

(c) The table doesn't consider $Z > 8$ but it does say that $P(Z \leq 8) \geq P(Z \leq 3.5) = 0.9998$, so $P(Z > 8) = 1 - P(Z \leq 8) < 0.0002$. For those curious, the exact value of $P(Z > 8)$ is

$$6.22 \times 10^{-16} = 0.000000000000000622.$$

(d) The table gives $P(Z < 0.5) = 0.6915$.

3.4:

(a) $N(\mu = 4313, \sigma = 583)$ for the men and $N(\mu = 5261, \sigma = 807)$ for the women.

(b)

$$Z_{Leo} = \frac{4948 - 4313}{583} \approx 1.09.$$

$$Z_{Mary} = \frac{5513 - 5261}{807} \approx 0.31.$$

Leo is 1.09 standard deviations slower than the mean for men between 30 and 34 years old. Mary is 0.31 standard deviations slower than the women between 25 and 29 years old.

(c) Mary was faster with respect to her group because her Z-score is lower (and lower Z-scores represent faster times).

(d) The percentage of athletes (in Leo's group) that finished faster than Leo is $P(Z < 1.09) = 0.8621$. So Leo finished faster than $1 - 0.8621 = 0.1379$.

(e) The percentage of athletes (in Mary's group) that finished faster than Mary is $P(Z < 0.31) = 0.6217$. So Mary finished faster than $1 - 0.6217 = 0.3783$.

(f) The answer to (b) is the same, but the other answers change. The percentiles are not the same for other distributions.

3.6:

(a) We want to find the position for the 5th percentile. In the table we look for 0.05 and figure out which Z attains it. It happens between -1.64 and -1.65, so let's take the midpoint $Z = -1.645$. Since $Z = -1.645$ and

$$Z = \frac{x - 4313}{583},$$

so we solve for x and we get

$$x = 583Z + 4313 = 583 \times (-1.645) + 4313 \approx 3354.$$

Therefore the top five percent, run the race in 3354 seconds or less.

(b) We want to find the position of the 90th percentile (since the 10 percent slowest start at the 90th percentile, because in racing, lower numbers represent faster times). We look up 0.9 in the table and see that it occurs when $Z = 1.28$. Then we have

$$x = 807 \times 1.28 + 5261 \approx 6294.$$

So the slowest ten percent (women in that group) finish the triathlon in 6294 seconds or more.

3.10:

(a) The Z-score is

$$Z = \frac{48 - 55}{6} \approx -1.17.$$

The probability that a randomly chosen 10 year old is shorter than 48 inches is $P(Z < -1.17) = 0.1210 = 12.1\%$.

(b)

$$Z_{60} = \frac{60 - 55}{6} \approx 0.83 \quad Z_{65} = \frac{65 - 55}{6} \approx 1.67.$$

The probability is

$$P(0.83 < Z < 1.67) = P(1.67) - P(0.83) = 0.9525 - 0.7967 = 0.1558.$$

(Note: We rounded the Z-scores to be able to use the tables. If we use the fractions and use a computer to calculate the actual probability without rounding, we get ≈ 0.1545. It's close to the answer we got with rounding.

(c) The Z score for the 10% tallest is a number A satisfying that $P(Z < A) = 0.9$. Using the the table we find $A \approx 1.28$. Therefore we have $Z = 1.28$ and we want to find the height h. We know

$$Z = \frac{h - 55}{6}.$$

We solve for h and using that $Z = 1.28$ we get

$$h = 6 \times (1.28) + 55 \approx 62.7 \text{ inches}.$$

(d)

$$Z = \frac{54 - 55}{6} \approx -0.17,$$

so we want $P(Z < -0.17) = 0.4325$.

3.13: The Z-score for overweight bags is:

$$Z = \frac{50 - 45}{3.2} \approx 1.56.$$

We're interested in $P(Z > 1.56)$. So we use the table to find $P(Z < 1.56) = 0.9406$. Therefore $P(Z > 1.56) = 1 - 0.9406 = 0.0594$. We get that the percent of airline passengers incurring the fee is 5.94%.

3.14:

(a) An IQ score of 132 is in the 98th percentile. To be in the 98th percentile, the Z-score must be about 2.055 (using the table). On the other hand

$$Z = \frac{132 - 100}{\sigma},$$

so

$$2.055 = \frac{32}{\sigma},$$

and hence

$$\sigma = \frac{32}{2.055} \approx 15.57.$$

(b) $\mu = 185$. High-cholesterol happens when $x > 220$ and it's in the 81.5th percentile. So the Z-score is around 0.9. Then we have

$$0.9 = \frac{220 - 185}{\sigma},$$

and hence

$$\sigma = \frac{35}{0.9} \approx 38.9.$$

3.16: Let A be the event that a student got more than 2100 and B be the event that a student got more than 1900. We want $P(A|B)$. We have

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}.$$

Since whenever A is true, B is also true, $P(A \text{ and } B) = P(A)$. So we actually just need to find the ratio of $P(A)/P(B)$. We need Z-scores. For the 2100 at SAT we have the Z-score:

$$Z = \frac{2100 - 1500}{300} = 2.$$

Therefore

$$P(A) = 1 - P(Z < 2) = 1 - 0.9772 = 0.0228.$$

For the 1900 SAT score, the Z-score is

$$Z = \frac{1900 - 1500}{300} = \frac{4}{3} \approx 1.33.$$

Therefore

$$P(B) = 1 - P(Z < 1.33) = 1 - 0.9082 = 0.0918.$$

Therefore

$$P(A|B) = \frac{0.0228}{0.0918} \approx .2484.$$

3.18:

(a) We have $\mu = 61.52$ and $\sigma = 4.58$, so the range within one standard deviation is from $61.52 - 4.58 = 56.94$ to $61.52 + 4.58 = 66.1$. The proportion of values in that range is $17/25 = 0.68$. It matches 68% as it would if it were a normal distribution.

The range within two standard deviations is $61.52 - 2 \times 4.58 = 52.36$ to $61.52 + 2 \times 4.58 = 70.68$. The proportion of values in that range in the data is $24/25 = .96$ (the only value outside the range is 73). Very close to 95%.

For three standard deviations the range is $61.52 - 3 \times 4.58 = 47.78$ to $61.52 + 3 \times 4.58 = 75.26$. The proportion of values in that range is $25/25 = 1$ which is very close to 99.7%.

Therefore the heights do follow the 68-95-99.7% Rule.

(b) Yes they do. The histogram is not as symmetric as it should be, but the Bell curve is a good approximation. The plot with the quantiles is very close to a line, which is what would occur if it were normally distributed.

3.30: The mean is $0.09 \times 15000 = 1350$. The standard deviation is $\sqrt{15000 \times 0.09 \times 0.91} \approx 35.05$. Therefore the $Z - score$ of 1500 is:

$$Z = \frac{1500 - 1350}{35.05} \approx 4.28.$$

The probability $P(Z > 4.28) = 1 - P(Z < 4.28) < 1 - .9998 = 0.0002$. It is extremely unlikely that 1500 or more will respond.

3.32:

(a) The probability that a teenager does not have arachnophobia is $1 - 0.07 = 0.93$. The probability that at least one of them has arachnophobia is 1 minus the probability that none of them have arachnophobia, so

$$1 - (0.93)^{10} = 0.516.$$

(b)

$$\binom{10}{2}(0.07)^2(0.93)^8 = 0.1234.$$

(c)

$$(0.93)^{10} + \binom{10}{1}(0.07)(0.93)^9 \approx 0.8483.$$

(d) It seems reasonable, in the sense that the chances of each tent having more than 1 teen with arachnophobia are small (0.1517), but if there are many teenagers, than you would need many tents and randomly assigning won't work too well. Here's an example, suppose you have 100 teenagers. Then, on average 7 of them will have arachnophobia. When you set up ten tents randomly, it is not that unusual for two of those 7 to end up in the same tent. In fact the chances that at least two of them end up in the same tent is

$$1 - \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4}{10^{10}} = 0.99994.$$

It is extremely likely that a tent will have at least two arachnophobes. So random assignment is a bad idea. The counselor, should ask the teens and assign arachnophobes to different tents.

3.33:

(a) $0.125 \times (1 - 0.125) \approx 0.1094$.

(b) $2 \times (0.125)(1 - 0.125) \approx 0.21875$.

(c) $\binom{6}{2}(0.125)^2(1 - 0.125)^4 \approx 0.1374$.

(d) $1 - (1 - 0.125)^6 \approx 0.5512$.

(e) $(1 - 0.125)^3(0.125) \approx 0.0837$.

(f) The probability that 2 or less children have brown eyes (out of 6) is

$$\binom{6}{0}(0.75)^0(0.25)^6 + \binom{6}{1}(0.75)^1(0.25)^5 + \binom{6}{2}(0.75)^2(0.25)^4 \approx 0.0376.$$

Yes, it would be unusual if only two children had brown eyes.

3.35:

Y	Probability of winning Y dollars
3	$\left(\frac{18}{38}\right)^3 \approx 0.1063$
1	$3\left(\frac{18}{38}\right)^2\left(\frac{20}{38}\right) \approx 0.3543$
-1	$3\left(\frac{18}{38}\right)^1\left(\frac{20}{38}\right)^2 \approx 0.3936$
-3	$\left(\frac{20}{38}\right)^3 \approx 0.1458$

3.36:

(a)

$$\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{9}{64} \approx 0.1406.$$

(b)

$$\binom{5}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 + \binom{5}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 = \frac{105}{1024} \approx 0.1025.$$

(c)

$$0.1025 + \left(\frac{1}{4}\right)^5 \approx 0.1035.$$