Homework 5 Solutions
Math 150

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4.4:

(a) The point estimate for the average is 171.1. For the median it is 170.3.

(b) For standard deviation: 9.4. For IQR: $177.8 - 163.8 = 14$.

(c) The 180cm person is not unusually tall because, the person is rougly one standard deviation from the mean. Not unusual. The 155cm is roughly 1.63 standard deviations away from the mean, so it's also not unusually short.

(d) No. I expect them to be close, but not exactly the same. There is some variability for different samples.

(e) The measure is the standard error and it comes out to:

$$SE = \frac{s}{\sqrt{n}} = \frac{9.4}{\sqrt{507}} \approx 0.417.$$

4.6:

(a) The sampling distribution.

(b) The sample size is 15 so it is not large enough that we can be sure the sampling distribution is normal. In this particular case, 14 out of 15, means that the distribution will have values at 11, 12, 13, 14, 15 (most of them at 13, 14, 15), so it will be left-skewed.

(c) The variability for this distribution is measured by the standard error. We're going to use $\bar{x} = 14/15$ as our point estimate:

$$SE = \frac{\sqrt{\left(\frac{14}{15}\right)\left(1 - \frac{14}{15}\right)}}{\sqrt{15}} = \frac{\sqrt{14}}{15\sqrt{15}} \approx 0.0644.$$

(d) This sampling distribution would have a smaller standard error. The new standard error would be the previous one times $\frac{\sqrt{15}}{\sqrt{25}}$.

4.8: The z-score that yields a 99% confidence interval is $z = 2.575$. Therefore the 99% confidence interval is

$$52 \pm 2.575 \times 2.4 = 52 \pm 6.18 = (45.82, 58.18).$$

4.10:

(a) False. The 99% includes numbers smaller than 50% as possibilities, so we cannot make that conclusion at the $\alpha = 0.01$ significance level.

(b) False. The standard error is not about how many users were included, it is an estimate for the variability in samples of the size used in the survey.

(c) False. More data shrinks the standard error.

(d) False. The "net" required to trap with 99% confidence is wider than the "net" required to trap with 90% confidence.

4.13:

(a) False, because the sample size is 64 and 64 is larger than 30. While the distribution might be skewed, it is probably not skewed enough to need a much larger sample size.

(b) False. We're certain what the average waiting time for these 64 patients is. We calculated it!

(c) True. That's what the confidence interval tells us.

(d) False. 95% of confidence intervals "trap" the parameter, but not 95% of samples lie inside this particular confidence interval. At least we can't say that.

(e) False. It's the other way around. The interval is wider because we need to be more sure the parameter is "trapped".

(f) True. The sample mean is the midpoint of the confidence interval (which is 137.5) and the margin of error is half of the length of the confidence interval (which is 9.5).

(g) False. We need to quadruple the sample size. That is because the standard error changes by the square root of the sample size. $\sqrt{4n} = 2\sqrt{n}$, while $\sqrt{2n} \neq 2\sqrt{n}$.

4.14:

(a) False. We know what the average spending for the sample is. The confidence interval extrapolates from that to all American adults.

(b) False. While the distribution is right-skewed, the sample size is large enough to make it irrelevant.

(c) False. If, magically, the sample mean is the population mean, then we can say that 95% of random samples have a sample mean in that range. However, the sample mean is unlikely to be the population mean, and in that case, a smaller percentage of random samples lie in this range.

(d) True. This is what the 95% confidence interval tells us.

(e) True. With a smaller confidence level, we will have a narrower confidence interval.

(f) False. We need to use a sample 9 times larger. This is because the "shrinkage" of the length of the confidence interval is proportional to $\sqrt{n}$ where n is the sample size.

(g) True. The margin of error is half of the length of the confidence interval.

4.16:

$$SE = 1.96 \times \frac{4.72}{\sqrt{5534}} = 1.96 \times 0.063\ldots \approx 0.1244.$$

Therefore the 95% confidence interval is

$$23.44 \pm 0.1244 = (23.2156, 23.5644).$$

That means we're 95% confident that the average age of woman at first marriage is between 23.22 and 23.56.

The sample size is so large that whether the distribution is skewed or not is irrelevant. We're also assuming that the data comes from a random sample of less than 10% of the population. Since 5536 is much smaller than 10% of the population of American women who have married, we can use the Central Limit Theorem to create the confidence interval.