Homework 6 Solutions
Math 150

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4.18:

(a) In words:

Null Hypothesis: The average calorie intake after menus displayed calorie counts is the same as the average calorie intake before menus displayed calorie counts, i.e., $\mu = 1100$.

Alternative Hypothesis: The average calorie intake after menus displayed calorie counts is the different than the average calorie intake before menus displayed calorie counts, i.e., $\mu \neq 1100$.

In symbols:

$$H_0 : \mu = 1100.$$
$$H_A : \mu \neq 1100.$$

(b) In words:

Null Hypothesis: The average Verbal Reasoning score is the same now as the average Verbal Reasoning score in 2004, i.e., $\mu = 462$.

Alternative Hypothesis: The average Verbal Reasoning score is different now than the average Verbal Reasoning score in 2004, i.e., $\mu \neq 462$.

In symbols:

$$H_0 : \mu = 462.$$
$$H_A : \mu \neq 462.$$

4.20: The null hypothesis and the alternative hypothesis are statements about the parameter μ not about the sample mean. So $\bar{x}$ should not be in the hypotheses. Another error is that the researcher is interested to find if there's an increase or a decrease, so the alternative hypothesis should be 2-sided.

For those curious, the correct hypotheses are:

$$H_0 : \mu = 23.44 \; years \; old.$$

$$H_A : \mu \neq 23.44 \; years \; old.$$

4.22:

(a) I think she is very likely to be wrong. The 95% confidence interval does not include \$100 as a plausible possibility.

(b) No, because the 90% confidence interval is narrower, so it doesn't include \$100 either.

4.24:

(a) The histogram is not very skewed and the sample size (36) is bigger than 30. The sample comes from a random sample and in a large city 36 is unlikely to be more than 10% of the population of gifted children, therefore the observations are approximately independent. Therefore, the conditions for inference are satisfied.

(b) The hypothesis are:

$$H_0 : \mu = 32.$$

$$H_A : \mu < 32.$$

The standard error is

$$SE = \frac{4.31}{\sqrt{36}} = \frac{4.31}{6} \approx 0.7183.$$

Therefore the standardized score for 30.69 is

$$z = \frac{30.69 - 32}{0.7183} = -\frac{1.31}{0.7183} \approx -1.82.$$

The p-value is approximately 0.0344. Therefore there is significant evidence to reject the null hypothesis.

(c) The p-value is the probability that a sample would be this "extreme" (or more) assuming the null hypothesis, i.e., assuming that the mean was 32 months.

With respect to this data, it is the probability that a sample of size 36 would have sample mean smaller than or equal to 30.69 assuming that the parameter has mean 32.

(d) For 90% confidence interval, the multiplier is 1.645. Therefore the 90% CI is:

$$30.69 \pm 1.645 \times 0.7183 = 30.69 \pm 1.1816.$$

Therefore the 90% confidence interval is $(29.5084, 31.8716)$.

(e) They do. The confidence interval doesn't contain 32, so both tests reject the null hypothesis in favor of the alternative hypothesis.

4.26:

(a)

$$H_0 : \mu = 100.$$

$$H_A : \mu \neq 100.$$

$$SE = \frac{6.5}{\sqrt{36}} = \frac{6.5}{6} \approx 1.083.$$

Then

$$z = \frac{118.2 - 100}{1.083} = \frac{18.2}{1.083} \approx 16.8.$$

The p-value is smaller than .0001. Therefore there is very strong evidence to reject the null hypothesis in favor of the alternative hypothesis.

(b) The multiplier is 1.645. Therefore the confidence interval is

$$118.2 \pm 1.645 \times 1.083 = 118.2 \pm 1.7815.$$

Therefore the 90% confidence interval is $(116.42, 119.98)$.

(c) Yes, the confidence interval doesn't include 100 as plausible. Therefore it comes to the same conclusion as the hypothesis test with p-values.

4.30:

(a)

Null Hypothesis: Sanitary regulations are being met at the restaurant.

Alternative Hypothesis: Sanitary regulations are not being met at the restaurant.

(b) Sanitary regulations are being met, but the inspector considers the restaurant is not meeting regulations and revokes the license.

(c) Sanitary regulations are not being met, but the inspector doesn't come to that conclusion.

(d) A Type 1 error is more problematic for the owner because the restaurant loses the license unfairly.

(e) A Type 2 error is more problematic for the diners because the restaurant is in gross violation of sanitary regulations, yet the restaurant remains open.

(f) As a diner, I would prefer strong evidence instead of very strong evidence. This would make a Type 2 error less likely.

4.32:

(a) **True**, because the 99% confidence interval is wider than the 95% confidence interval.

(b) **False**, because the significance level is the probability of making a Type 1 error. One possible correct statement is: "Decreasing the significance level (α) will decrease the probability of making a Type 1 Error." There are other correct ways of making a correct statement regarding α and the type of error.

(c) **False**, when we fail to reject the null hypothesis, it doesn't mean we accept the null hypothesis, it means that the null hypothesis is plausible. There's no phrasing of this statement that keeps the gist of the statement while transforming it from False to True.

(d) **True**, because that's the definition of the power of a test.

(e) **True**, because the larger the sample size, the more likely it is we detect small differences.

4.36:

(a) Left skewed. Because the max score is 100 and the mean is already at 74, there is a barrier on the right preventing a long right tail. At the same time, there are scores to the left of 20, so the left tail goes longer. Furthermore, the median is higher than the mean, suggesting that the mean is pulled to the left.

(b) More students scored above 70 because the median is 74, so at least 50% of the students scores 74 or more.

(c) We cannot because the distribution is not normal.

(d) Since 40 is a large enough sample, we can use the central limit theorem to get an approximation. The Standard Error is

$$SE = \frac{10}{\sqrt{40}} \approx 1.581.$$

Therefore the standardized score for 75 is

$$z = \frac{75 - 70}{1.581} \approx 3.16.$$

Using the normal distribution, the probability that a random sample of 40 students is above 75 is approximately $1 - P(z < 3.16) = 1 - 0.9992 = 0.0008$.

(e) It make the standard deviation $\sqrt{2} \approx 1.41$ larger.

4.40:

(a)

$$z = \frac{10500 - 9000}{1000} = \frac{1500}{1000} = 1.5.$$

Therefore we want $P(z > 1.5) = 1 - P(z < 1.5) = 1 - 0.9332 = 0.0668$.

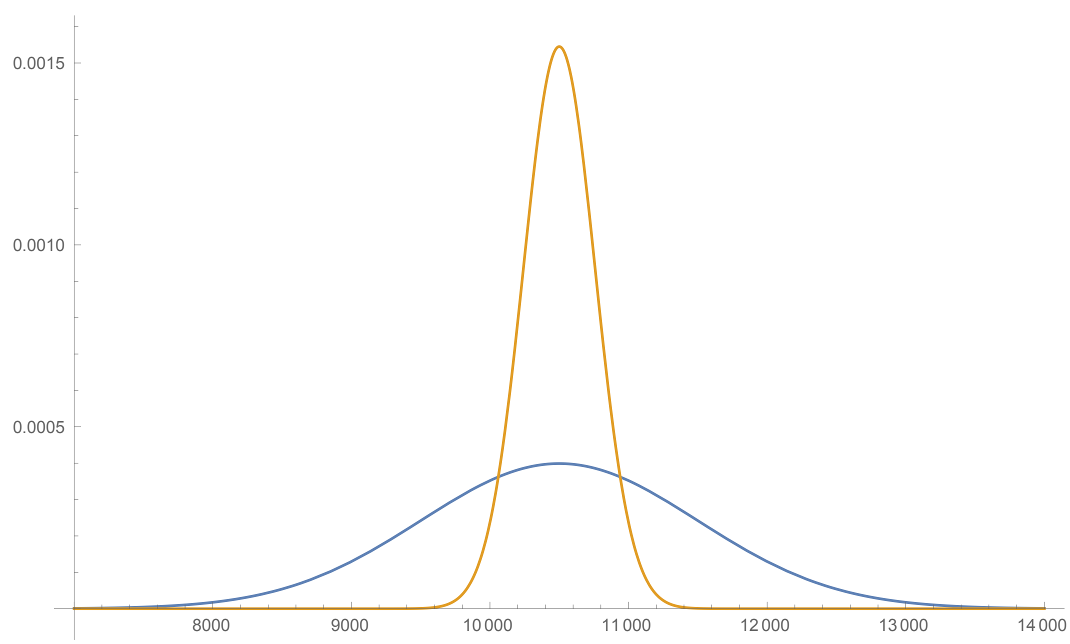
(b) It is normal with mean 9000 hours and standard deviation $\dfrac{1000}{\sqrt{15}} \approx 258.2$

(c)

$$z = \frac{10500 - 9000}{258.2} \approx 5.81.$$

Since $z > 3.5$, the probability is less than .0001.

(d)



(e) No, because the calculations assume we have a normal distribution.

4.41:

(a) Since the distribution is skewed we can't calculate using the normal distribution. But we can estimate it using the histogram. There are roughly 500 songs that last 5 minutes or more. There are 3000 songs, so the probability is about

$$\frac{500}{3000} \approx 0.167.$$

(b) 15 is a small sample size, so one could argue that we can't calculate this probability. That is a valid answer.

Since the distribution is not heavily skewed, 15 is probably a large enough sample size to estimate the probability using the normal distribution. Let's calculate it under this assumption. We want to find the probability that a random sample of 15 songs has average length $60/15 = 4$. Since the standard deviation for the length of the songs is 1.63, then the standard error is

$$SE = \frac{1.63}{\sqrt{15}} \approx 0.421.$$

Then the z-score is:

$$z = \frac{4 - 3.45}{0.421} \approx 1.31.$$

The probability of having a playlist last for the entire run is $1 - P(zM1.31) = 1 - .9049 = 0.0951$.

(c) This time the sample size is 100, so we can definitely approximate this using the normal distribution.

$$SE = \frac{1.63}{\sqrt{100}} = 0.163.$$

With 100 songs lasting at least 6 hours, the average length of the song should be greater than or equal to 3.6 (because there are 360 minutes in 6 hours). Then

$$z = \frac{3.6 - 3.45}{0.163} \approx 0.92.$$

Therefore the probability is $1 - P(z < 0.92) = 1 - .8212 = 0.1788$.