Homework 7 Solutions
Math 150

Enrique Treviño

5.1:

(a) df $= 5$, and $t^* = 2.02$. To find t^*, we look at table $B.2$.

(b) df $= 20$, and $t^* = 2.53$.

(c) df $= 28$, and $t^* = 2.05$.

(d) df $= 11$, and $t^* = 3.11$.

5.2: The solid line is the normal distribution. The dashed line is the t-distribution with 5 degrees of freedom. The dotted line is the t-distribution with 1 degree of freedom. The reason is that the less degrees of freedom, the 'thicker' the tails. Clearly the thickets tail is for the dotted line, followed by the dashed line and the solid line (in that order).

5.4:

(a) df $= 25$. Using the table with row 25 we see that 2.485 is between 2.06 and 2.49, so the one sided tail has area between 0.010 and 0.025. It's much closer to 0.010 since $2.485 \approx 2.49$, but it is still larger than 0.010, therefore the null hypothesis is not rejected at $\alpha = 0.01$.

(b) df $= 17$. $0.5 < 1.33$, so the p-value is larger than 0.100. Therefore the null hypothesis is not rejected at $\alpha = 0.01$.

5.8:

(a) The sample is smaller than 10% of the population. We are not told whether the 14 users were randomly sampled. Assuming they are, then the observations are independent. The histogram is mostly symmetric, so the near-normal condition is satisfied. Therefore it is reasonable conditioned on the sample being a simple random sample.

(b) Assuming that the observations are independent and nearly normal, we can proceed finding a t-score and then getting a p-value.

$$t = \frac{53.3 - 50}{\left(\frac{5.2}{\sqrt{14}}\right)} \approx \frac{3.3}{1.38976} \approx 2.37.$$

Given df $= 13$, and that it's a two-tails question, using table $B.2$ we find that $2.16 < 2.37 < 2.65$, so the p-value is between 0.02 and 0.05. Therefore, we reject the null hypothesis in favor of the alternative, i.e., we have strong evidence that the 2012 Prius does not get 50 MPG (the evidence suggests, it gets more miles per gallon).

(c) We have df $= 13$. Therefore $t^*_{13} = 2.16$ (we find this number using table $B.2$ by going to row df $= 13$ and column "two tails$= 0.05$"). Therefore the 95% confidence interval is

$$53.3 \pm (2.16)\left(\frac{5.2}{\sqrt{14}}\right) = 53.3 \pm 3.00.$$

Therefore it is $(50.3, 56.3)$.

5.12:

(a) In words, the hypothesis are:

Null Hypothesis: The average lead concentration for police officers is the same as the average for people not exposed to automobile exhaust fumes, i.e., $mu = 35$.

Alternative Hypothesis: The average lead concentration for police officers is greater than the average for people not exposed to automobil exhaust fumes, i.e., $\mu > 35$.

In symbols:

$\mathbf{H_0}$: $\mu = 35$.

$\mathbf{H_A}$: $\mu > 35$.

(b) We are not told whether the police officers are randomly sampled. It's unclear whether the independence condition is satisfies. The researchers are interested in lead exposure in general, so testing police officers is a convenience sample, not a random sample. The near-normality condition is hard to check without more data, but the sample size is 52, so it's probably big enough to hold.

(c) Under the assumption that the police officers were randomly sampled, we can test under the context of whether police officers in urban settings have higher lead concentration than people living in nearby suburbs. In other words, the study cannot be generalized to the population as a whole. With that being said, let's test it.

$$t = \frac{124.32 - 35}{\left(\frac{37.74}{\sqrt{52}}\right)} \approx \frac{89.32}{5.2336} \approx 17.07.$$

The t-score is enormous, so the p-value is minuscule. Therefore there is enough evidence to reject the null hypothesis in favor of the alternative hypothesis.

(d) No. The p-value is very very very small. Much smaller than .005 (which is the size it would have to be to be included in the 99% confidence interval).

5.14:

(a) The margin of error is $t^* \times SE$. For a 90% confidence interval, we have that t^* is the value in the second column of $B.2$. It depends on the degrees of freedom which would be $n - 1$. We know that $t^* \geq 1.28$. We also know $SE = 250/\sqrt{n}$. So we have

$$\begin{aligned} \frac{250t^*}{\sqrt{n}} &= 25 \\ \frac{250}{25}t^* &= \sqrt{n} \\ 100(t^*)^2 &= n. \end{aligned}$$

Therefore

$$n \geq 100 \times (1.28)^2 \approx 163.84.$$

If df $= 150$ or df $= 200$, then $t^* = 1.29$, therefore we need

$$n \geq 100 \times (1.29)^2 \approx 166.41.$$

Therefore if $n \geq 167$, then we can get a margin of 25 for a 90% confidence interval.

(b) It has to be larger, because t^* would be bigger for a 99% confidence interval.

(c) The idea is similar, but since it is a 99% confidence interval we have $t^* \geq 2.58$. That means that

$$n \geq 100 \times (2.58)^2 \approx 665.64.$$

Given that the table only gets to 500, we'll have to settle for using $t^* \approx 2.59$ for the large n, so we get

$$n \geq 100 \times (2.59)^2 \approx 670.81.$$

Therefore we need at least 671 to be sampled.

5.15:

(a) Two sided test because we want to compare the years. We don't have a reason to suspect it will be higher or lower.

(b) We should use a paired test because we can compare the same cities to themselves.

(c) t-test because the sample is small and the population standard deviation is unknown.

5.16:

(a) True. (Note: The sentence is ambiguous, if one interprets "each pair" as every possible pair of observations from the data set, then it the statement would be false. Given the context it is more reasonable to interpret "each pair" as the pairs from the paired data).

(b) True. For a matched pair analysis, we need to pair up each observation from one data set with an observation from the other data set. They have to have the same sample sizes.

(c) True.

(d) False. We subtract each pair of "paired" observations. (Note: If the inference is done with this technique, then the data has the same point estimate for the mean of the differences, but it has a different standard deviation).

5.18:

(a) Paired.

(b) Paired. The natural correspondence is an item in one store versus the same item in the other store.

(c) Not paired. There is not a natural correspondence from an individual student in one school to an individual student in the other.

5.20:

(a) The relevant data is the histogram, since it's the distribution of the paired differences. There seems to be more negative differences than positive differences. The proportion is slight, without knowing the actual numbers, one can't conclusively say it's clear. (Note: If one looks at the box plots, the box plots don't have clear evidence one way or another because the medians and quartiles are about the same. However, looking at the box plots is not relevant for this question.)

(b) No.

(c) In words, the hypothesis are:

Null Hypothesis: The average scores in the reading exams and in the writing exams are the same, i.e., $\mu_{\text{read}} = \mu_{\text{write}}$. Therefore $\mu_{\text{diff}} = \mu_{\text{read}} - \mu_{\text{write}} = 0$.

Alternative Hypothesis: The average scores in the reading exams and in the writing exams are different, i.e., $\mu_{\text{read}} \neq \mu_{\text{write}}$. Therefore $\mu_{\text{diff}} = \mu_{\text{read}} - \mu_{\text{write}} \neq 0$.

In symbols:

$\mathbf{H_0}$: $\mu_{\text{diff}} = 0$.

$\mathbf{H_A}$: $\mu_{\text{diff}} \neq 0$.

(d) We have a simple random sample (it's from a survey, so it might not be random enough) and the sample is of less than 10% of the population. So we may assume the observations are independent. The distribution of the differences is nearly normal. It's a little skewed, but with $n = 200$, the skewness does not impact the results.

(e)

$$t = \frac{-0.545 - 0}{\left(\frac{8.887}{\sqrt{200}}\right)} \approx \frac{-0.545}{0.6284} \approx -.867.$$

Given that it's a two-sided test, the p-value with df $= 199$ for that t-score is bigger than 0.2, therefore there is not evidence to reject the null hypothesis.

(f) Since we didn't reject the null hypothesis, the possible error is a Type 2 error (failing to reject the null when the null is false). In the context, a Type 2 error means that there is a difference in the scores of students in the reading and writing sections but that we didn't detect it.

(g) Yes, because the null hypothesis is not rejected, so 0 is a plausible value.

5.22:

(a) The sample size is 200, so df $= 199$. The table includes values for df $= 150$ and df $= 200$. For a 95% confidence interval, the row of df $= 150$ suggests using $t^* = 1.98$, for df $= 200$, it suggests $t^* = 1.97$. We'll err on the side of caution and use 1.98 (but using 1.97 is also acceptable):

$$\bar{x} \pm t^* \times SE = -0.545 \pm (1.98) \times \left(\frac{8.887}{\sqrt{200}}\right) \approx -0.545 \pm 1.244.$$

Therefore an approximation of the confidence interval is $(-1.789, 0.699)$.

Note: If you use $t^* = 1.97$, then the confidence interval comes out to $(-1.783, 0.693)$. Using statistical software, it calculates the confidence interval as $(-1.784, 0.694)$.

(b) We are 95% confident that the average difference between the reading portion of the test and the writing portion of the test is between -1.789 and 0.699.

(c) No, because 0 is a plausible average difference

5.26: It doesn't satisfy that the observations are independent. The winners from year to year are not independent (for example, Meryl Streep has been a contender for decades). It also misses out on the variable of "year". A paired study would be much better to study this question (albeit, it would also have some issues with the problem of independence).

5.28: The hypothesis are:

Null Hypothesis: The average standardized prices of 0.99 carat diamonds is the same as the average standardized prices for 1 carat diamonds.

Alternative Hypothesis: The average standardized prices of 0.99 carat diamonds is less than the average standardized prices for 1 carat diamonds.

In symbols (letting μ be the average of 1 carat diamond standardized prices minus the average standardized prices of the 0.99 carat diamonds):

$\mathbf{H_0}$: $\mu = 0$.

$\mathbf{H_A}$: $\mu > 0$.

The observations are independent since they come from simple random samples of less than 10% of the population (there are many more than 230 diamonds, so 23 is less than 10%). The boxplots suggest that the distributions are a skewed. But they are not very skewed, so giving the sample size of 23, it is reasonable to assume near-normality.

$$SE = \sqrt{\frac{13.32^2}{23} + \frac{16.13^2}{23}} \approx \sqrt{\frac{437.5993}{23}} \approx \sqrt{19.026} \approx 4.3619.$$

Therefore the t-score is

$$t = \frac{56.81 - 44.51}{4.3619} = \frac{12.3}{4.3619} \approx 2.82.$$

We have df $= 22$ (we are erring on the safe side by taking one less than the minimum of the sample sizes). Using table $B.2$ we have that the p-value for the one-sided test is approximately 0.005. We have enough evidence to reject the null hypothesis in favor of the alternative hypothesis. That is, we have evidence that suggests that the price of 1 carat diamonds is inflated disproportionally to the number of carats.

5.30: We are erring on the safe side and using df $= 22$. Since it's a 95% confidence interval, then $t^* = 2.07$ (using table $B.2$). We also know from work on the previous exercise that $SE \approx 4.3619$. Therefore the confidence interval is

$$12.3 \pm 2.07 \times 4.3619 \approx 12.3 \pm 9.03.$$

Therefore the confidence interval is $(3.27, 21.33)$.

5.32:

$$SE = \sqrt{\frac{3.58^2}{26} + \frac{4.51^2}{26}} = \sqrt{\frac{33.1565}{26}} = \sqrt{1.27525} \approx 1.1293.$$

Therefore

$$t = \frac{19.85 - 16.12}{1.1293} = \frac{3.73}{1.1293} \approx 3.30.$$

Using df $= 25$ and that $3.30 > 2.79$, we can see that the p-value is smaller than 0.01. Therefore we have strong evidence to reject the null hypothesis in favor of the alternative hypothesis. That is, there is evidence that suggests that manual transmission cars are more fuel efficient than automatic cars.

Note: This study would have been better if we had paired data. Comparing the same car with a manual transmission versus an automatic transmission.

5.35:

$$SE = \sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}} = \sqrt{\frac{2730.97}{22}} = \sqrt{124.135} \approx 11.1416.$$

Therefore

$$t = \frac{52.1 - 27.1}{11.1416} = \frac{25}{11.1416} \approx 2.24.$$

Using df $= 21$, using table $B.2$ we see that because $2.07 < 2.24 < 2.50$, then the p-value is between .02 and .05 (it is a two sided test, so we use the numbers from two tails). Therefore we have enough evidence to reject the null hypothesis in favor of the alternative hypothesis, i.e., there is strong evidence that the average food intake is larger for people eating while playing solitaire.