MATH 150 MIDTERM #2
March 17, 2016

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

• You may NOT use a calculator (except for the one provided).

• You may NOT use your smartphone for any purpose. Please silence it or turn it off.

• Show all of your work.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
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<td>1</td>
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<td>Total:</td>
<td>72</td>
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</table>
1. What percent of a standard normal distribution $N(\mu = 0, \sigma = 1)$ is found in each region?
   (a) [2 points] $Z < -1.35$
   \[ \boxed{1.0885} \]
   (b) [2 points] $Z > 1.48$
   \[ 1 - .9306 = \boxed{.0694} \]
   (c) [2 points] $Z > 2$
   \[ 1 - .9772 = \boxed{.0228} \]
   (d) [2 points] $-0.4 < Z < 1.5$
   \[ .9332 - .3446 = .5886 \]

2. Sophia who took the Graduate Record Examination (GRE) scored 160 on the Verbal Reasoning section and 157 on the Quantitative Reasoning section. The mean score for Verbal Reasoning section for all test takers was 151 with a standard deviation of 7, and the mean score for the Quantitative Reasoning was 153 with a standard deviation of 7.67. Suppose that both distributions are nearly normal.
   (a) [2 points] What is Sophia’s Z-score on the Verbal Reasoning section?
   \[ \frac{160 - 151}{7} = \frac{9}{7} \approx 1.286 \approx \boxed{1.29} \]
   (b) [2 points] What is Sophia’s Z-score on the Quantitative Reasoning section?
   \[ \frac{157 - 153}{7.67} = \frac{4}{7.67} \approx \sqrt[4]{0.52} \]
   (c) [2 points] Relative to others, which section did she do better on?
   Verbal Reasoning.
(d) [2 points] Find her percentile score for the Verbal Reasoning section.

\[ z = 1.29 \text{ so her percentile is } 90.15 \%
\]

So \(90^{th}\) percentile

(c) [2 points] Find her percentile score for the Quantitative Reasoning section.

\[ z = 0.52 \text{ Area } 0.6985 \]

So \(69^{th}\) percentile

(f) [2 points] What percent of the test takers did better than her on the Verbal Reasoning section?

\[ 1 - 0.9015 = 0.0985 \]

\[ 9.85\% \]

(g) [2 points] What percent of the test takers did better than her on the Quantitative Reasoning section?

\[ 1 - 0.3015 = 0.6985 \]

\[ 30.15\% \]

(h) [2 points] Explain why simply comparing raw scores from the two sections could lead to an incorrect conclusion as to which section a student did better on.

Because it doesn't consider the variance.

(i) [2 points] Find the score of a student who scored in the 80th percentile on the Quantitative Reasoning section.

Find \(z\) st area is 0.8. \(z = 0.84\).

Then \[ x = 153 + (0.84)(7.67) = 159.44 \]

The student would need a 160 score to be in the 80th percentile.

(j) [2 points] Find the score of a student who scored worse than 70% of the test takers in the Verbal Reasoning section.

That means the percentile is 30th.

So \(z = -0.525\).

Then \[ x = 151 + (-0.525)(7) = 147.325 \]

So \(147\) is the 30th percentile score.
3. A college counselor is interested in estimating how many credits a student typically enrolls in each semester. The counselor decides to randomly sample 100 students by using the registrar’s database of students. The histogram below shows the distribution of the number of credits taken by these students. Sample statistics for this distribution are also provided.

\[
\begin{align*}
\text{Min} &= 8 \\
\text{SD} &= 1.91 \\
\text{Q1} &= 13 \\
\text{Q3} &= 15 \\
\text{Median} &= 14 \\
\text{Mean} &= 13.65 \\
\text{Max} &= 18
\end{align*}
\]

(a) [2 points] What is the point estimate for the average number of credits taken per semester by students at this college?

\[
13.65
\]

(b) [2 points] What is the point estimate for the median?

\[
14
\]

(c) [2 points] What is the point estimate for the standard deviation of the number of credits taken per semester by students at this college?

\[
1.91
\]

(d) [2 points] What is the point estimate for the IQR?

\[
15 - 13 = 2
\]

(e) [2 points] Is a load of 16 credits unusually high for this college? Explain your reasoning.

*Hint:* Observations farther than two standard deviations from the mean are usually considered to be unusual.

\[
\frac{16 - 13.65}{1.91} \approx \frac{2.35}{1.91} \approx 1.23.
\]

It's not unusual.

(f) [2 points] What about 18 credits?

\[
\frac{18 - 13.65}{1.91} \approx \frac{4.35}{1.91} \approx 2.28.
\]

It is unusual.
(g) [2 points] The sample means given above are point estimates for the mean number of credits taken by all students at that college. The standard error $SE$ is the measure we use to quantify the variability of this estimate. Find the standard error in the original sample.

$$SE = \frac{1.91}{\sqrt{100}} = 0.191$$

(h) [2 points] The college counselor takes another random sample of 100 students and this time finds a sample mean of 14.20 units. Should she be surprised that this sample statistic is slightly different than the one from the original sample? Explain your reasoning.

$$z = \frac{14.2 - 13.65}{0.191} \approx 2.88.$$ Yes, the counselor should be surprised. It is very far from the previous sample.

4. A hospital administrator hoping to improve wait times decides to estimate the average emergency room waiting time at her hospital. She collects a simple random sample of 64 patients and determines the time (in minutes) between when they checked in to the ER until they were first seen by a doctor. A 95% confidence interval based on this sample is (128 minutes, 147 minutes), which is based on the normal model for the mean. Determine whether the following statements are true or false.

(a) [2 points] This confidence interval is not valid since we do not know if the population distribution of the ER wait times is nearly Normal.

FALSE. Sample size is large enough.

(b) [2 points] We are 95% confident that the average waiting time of these 64 emergency room patients is between 128 and 147 minutes.

FALSE. We know the waiting time for these patients.

(c) [2 points] We are 95% confident that the average waiting time of all patients at this hospital’s emergency room is between 128 and 147 minutes.

TRUE.

(d) [2 points] 95% of random samples have a sample mean between 128 and 147 minutes.

FALSE. The confidence interval depends on the sample.

(e) [2 points] A 99% confidence interval would be narrower than the 95% confidence interval since we need to be more sure of our estimate.

FALSE. The higher the confidence, the wider the interval.

(f) [2 points] The margin of error is 5.5 and the sample mean is 137.5.

TRUE.

(g) [2 points] In order to decrease the margin of error of a 95% confidence interval to half of what it is now, we would need to double the sample size.

FALSE. We would need to quadruple the size.
5. The National Survey of Family Growth conducted by the Centers for Disease Control gathers information on family life, marriage and divorce, pregnancy, infertility, use of contraception, and men's and women's health. One of the variables collected on this survey is the age at first marriage. The histogram below shows the distribution of ages at first marriage of 5,534 randomly sampled women between 2006 and 2010. The average age at first marriage among these women is 23.44 with a standard deviation of 4.72.

(a) [2 points] Estimate the average age at first marriage of women using a 95% confidence interval.

\[ \bar{x} = 23.44 \pm (1.96) \left( \frac{4.72}{\sqrt{5534}} \right) \]

\[ \approx 23.44 \pm 0.124 \]

\[ \Rightarrow (23.316, 23.564) \]

(b) [2 points] Estimate the average age at first marriage of women using a 99% confidence interval.

\[ \bar{x} = 2575 \]

\[ 23.44 \pm (2.575) \left( \frac{4.72}{\sqrt{5534}} \right) \]

\[ = 23.44 \pm 0.163 \]

\[ \Rightarrow (23.277, 23.603) \]

(c) [2 points] Explain why the confidence intervals are valid.

Because the sample size is very big, yet the observations are independent (because 5534 women are less than 10% of the pop. of women).
6. A dreidel is a four-sided spinning top with the Hebrew letters nun, gimel, hei, and shin, one on each side. Each side is equally likely to come up in a single spin of the dreidel. Suppose you spin a dreidel three times. Calculate the probability of getting

![Photo of a dreidel](image)

**Figure 1: Photo by Staccabees, cropped, CC BY 2.0 license**

(a) [2 points] at least one nun?

\[
P(\text{at least one } \text{nun}) = (0.75)^3 = 0.421875 = \frac{27}{64}
\]

or

\[
1 - \frac{27}{64} = \frac{37}{64}
\]

(b) [2 points] exactly 2 nunns?

\[
3 \left( \frac{1}{4} \right)^2 \left( \frac{3}{4} \right) = \frac{9}{64} \approx 0.140625
\]

(c) [2 points] exactly 1 hei?

\[
3 \left( \frac{1}{4} \right) \left( \frac{3}{4} \right)^2 = \frac{27}{64} = 0.421875
\]

(d) [2 points] at most 2 gimesls?

\[
P(\text{at most 2}) = 1 - \frac{1}{64} = \frac{63}{64} \approx 0.984375
\]