Homework 8
Math 329: Number Theory

April 3, 2018

1 Easy

Problem 1. For all residues x modulo 11, evaluate $x^3 \bmod 11$ (for example, for mod 5, we'd have $1^3 \equiv 1 \bmod 5, 2^3 \equiv 3 \bmod 5, 3^3 \equiv 2 \bmod 4, 4^3 \equiv 4 \bmod 5$).

Problem 2. Show 2 is a primitive root modulo 11.

Problem 3. Evaluate $2^3, 2^6, \ldots, 2^{30} \bmod 11$.

Problem 4. Evaluate $2^{10} \bmod 121$.

Problem 5. Show that 2 is a primitive root modulo 121.

2 Medium

Problem 6. Let p be an odd prime. Prove that if a is a quadratic residue modulo p, then a is not a primitive root modulo p.

Problem 7. Let a and g be primitive roots modulo p (where p is an odd prime). Prove that ag is not a primitive root modulo p. (Hint: Write a as a power of g)

Problem 8. Suppose there is a primitive root g modulo n. Prove that there are $\phi(\phi(n))$ primitive roots modulo n. (Hint: Suppose a is a primitive root modulo n. Show that $a \equiv g^i \bmod n$ for some i satisfying that $(i, \phi(n)) = 1$.)

Problem 9. Suppose n is a squarefree positive integer, i.e., there is no integer k such that $k^2|n$. Prove that the following two are equivalent:

(a) For all integers a satisfying $\gcd(a, n) = 1$, $a^{n-1} \equiv 1 \bmod n$.

(b) For all prime divisors p of n, we have $p - 1|n - 1$.

Hint: Consider a prime divisor $p|n$. Let g be a primitive root of p. Show that (a) implies $p - 1|n - 1$.

3 Hard

Problem 10. Let p be an odd prime. Suppose g is a primitive root modulo p. Show that there is an integer x such that $g^x \equiv x \bmod p$.