Homework 6

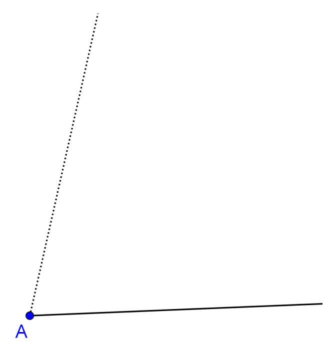
Book problems

1. Show that the regular 9-gon is not constructible with straightedge and compass, but that the regular 20-gon is.

2. Can a cube be constructed with three times the volume of a given cube?

   Ancient Geometry chapter problems

3. Given a point A and a line ℓ through A. Describe how you would create, using only straightedge and compass, a line k that goes through A that satisfies that the small angle between k and ℓ is $75°$. In the figure below, the dotted line is what k should be and ℓ is the solid line.



4. Let n be a positive integer. Show you can find a length of $\sqrt{n}$ using only straightedge and compass.

5. Given a regular n-gon and a regular m-gon satisfying that n and m are relatively prime, show that you can create a regular nm-gon using only straightedge and compass.

   Origami Paper (inspired) Problems

6. In the book it's proved that a constructible number α with straightedge in compass must satisfy that $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 2^n$ for some nonnegative integer n. Using that proof and the origami paper's claim that roots of an arbitrary cubic equation $x^3 + ax + b$ are *origami-constructible*, to show that if α is origami-constructible, then $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 2^k 3^m$ for some nonnegative integers k, m.

7. Show that regular 9-gons and regular 20-gons can be constructed with origami folds.

8. Suppose that p is prime. For a regular p-gon to be Euclidean constructible, then the roots of $x^p - 1$ must be constructible. The roots of $x^{p-1} + x^{p-2} + \cdots + x + 1$ together with $x = 1$ form a regular p-gon. They would need to be constructible. Since $x^{p-1} + \cdots + x + 1$ is irreducible, that means the degree of a root of this is $p - 1$. Using this prove that $p = 2^{2^n} + 1$ for some nonnegative integer n.

9. Suppose that p is prime and that we want to analyze when the regular p-gon can be constructed using origami. Find all $p \leq 100$ for which the regular p-gon can be constructed with origami.

Bonus

10. Given two points A and B, one can find using only compass (without the straightedge) a point C such that $\triangle ABC$ is equilateral. One can also find points to make an hexagon using only compass. Prove or disprove that you can find, using only a compass, points C and D such that $ABCD$ is a square.

11. Given a circle of area A, show that you can construct with compass alone, a circle of area nA for any positive integer n.