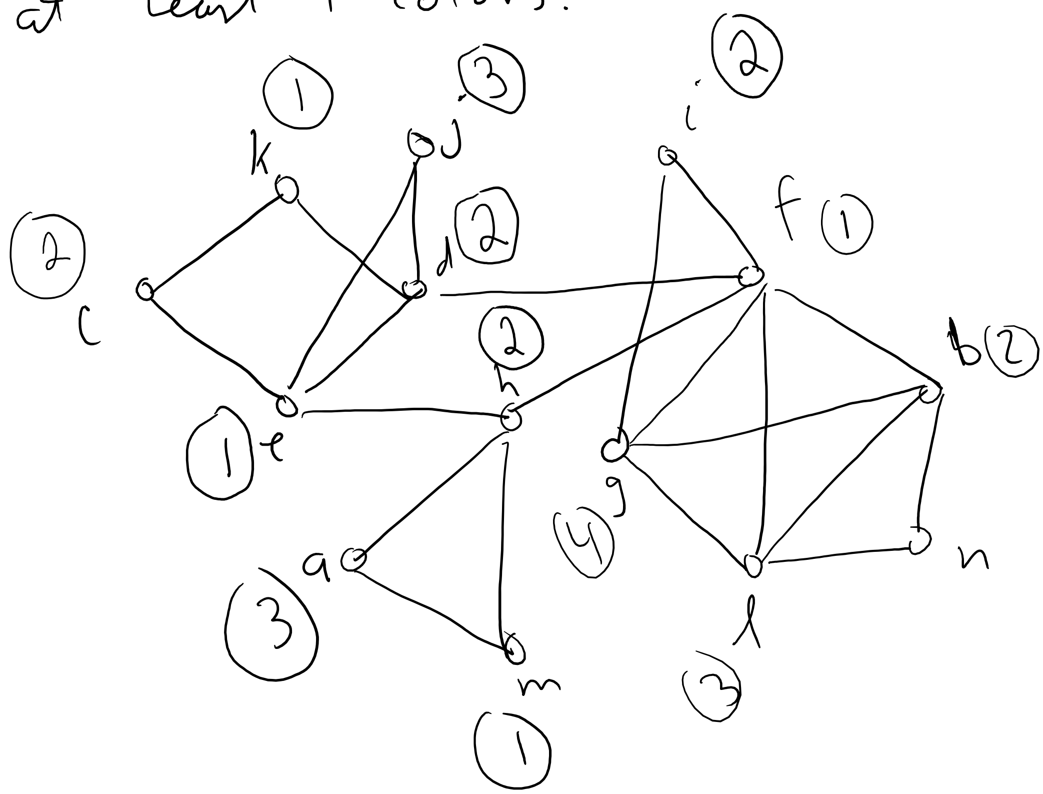
Homework 5 Solutions

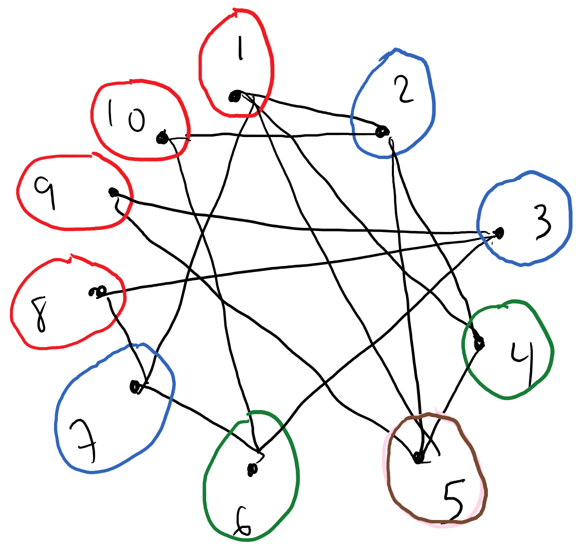
14

Since f, b, l, g forms a K_4, we need at least 4 colors.



4 colors works, so $\chi(G) = 4$
(I used 1, 2, 3, 4 to represent colors)

(15)



We want to find the chromatic number (since you could think of a color as a storeroom).

Because of 1, 2, 4, 5 we need at least 4 colors.

Using red, blue, green and brown we show 4 colors suffice. Therefore we need 4 storerooms (and no more, no less).

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(17) It's 2. You need at least 2 because the tree is connected, so it has at least one edge (which requires 2 colors). Since trees contain

no cycles, they contain no odd cycles, so they are 2-colorable. So $\chi(T)=2$.

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(21) $n_3 = 5$.

$$n_{t+1} = \binom{t(n_t-1)+1}{n_t} n_t + t(n_t-1)+1$$

For each n_t-subset of I, you make a copy of G_t

$|I| = t(n_t-1)+1$. So we have $\binom{|I|}{n_t} n_t + |I|$.

For example $n_4 = 5\binom{13}{5} + 13 = 6448$

$$n_5 = 6448\binom{25789}{6448} + 25789 \approx 1.2331 \times 10^{6300}$$

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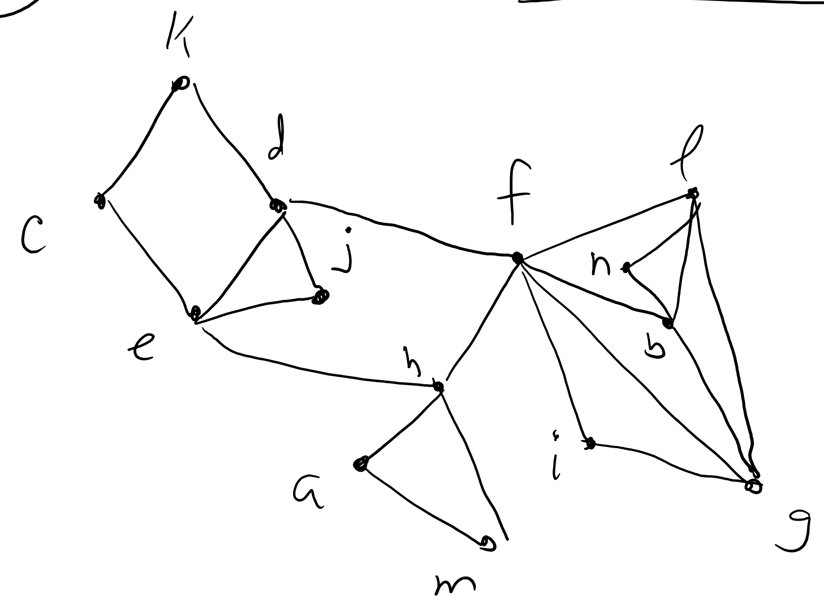
(22) $n_3 = 5$

$n_{t+1} = 2n_t + 1$

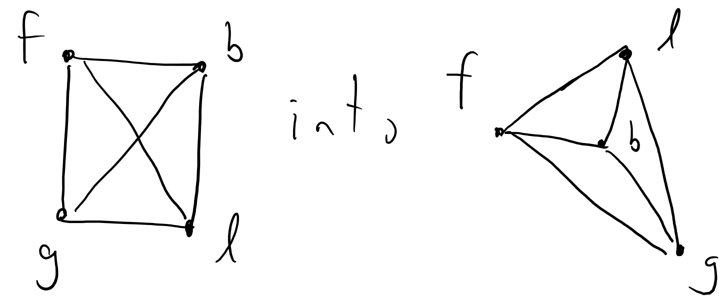
so $n_4 = 11$, $n_5 = 23$, $n_6 = 47$, $n_7 = 95$, ...

It is clear

(28) It is planar



(one useful trick is to transform



)

(32) Suppose every vertex is incident to at least 6 edges. Therefore every vertex has degree ≥ 6.

Then $|E| = \frac{1}{2} \sum_{v \in V} \deg(v) \geq 3n$

But by Theorem 5.33, planar graphs have

at most $3n-6$ edges. Contradiction!

Therefore one vertex has degree at most 5.

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33) Cannot be the degree sequence of any graph:

c) $n=7$ $(5,4,4,3,2,1,0)$

The sum of the degrees is odd, which is impossible in a graph.

Two sequences that could be planar graphs:

Note $|E| \leq 3n-6$ so $\sum \deg(v) \leq 6n-12$.

e) has sum of degrees $= 18 \leq 6(6)-12 = 24$ ✓

a) has sum $= 18 \leq 60-12 = 48$ ✓

So $\boxed{a, e}$

The one sequence that could be a tree:

Trees are planar so it's either a or e.

e can't be a tree. one vertex is connected to all others, so if any other vertex has degree ≥ 1 then we get a cycle. Therefore the only one that can be a tree is (a).

The one sequence that is Eulerian

(b) It's the only one where all vertices have even degree.

The one sequence that **must** be Hamiltonian

(d) Every vertex has degree $\geq \lceil \frac{10}{2} \rceil = 5$.

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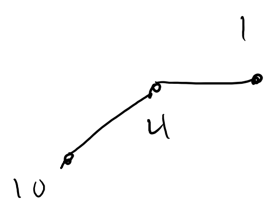
(37)



First leaf is 2 so 1

Next leaf is 3 connected to 4 — 1 4

Next leaf is 5 " " 6 — 1 4 6

Next 6 " " 9 — 1 4 6 9

7 $\to$ 4 — 1 4 6 9 4

8 $\to$ 9 — 1 4 6 9 4 9

9 $\to$ 1 — 1 4 6 9 4 9 1

1 $\to$ 4 — 1 4 6 9 4 9 1 4

So $\boxed{14694914}$

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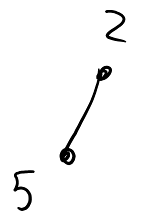
41. The tree is from $\{1,2,3,4,5,6,7\}$

The initial leafs are 5, 6, 7

code

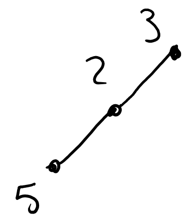
$\underline{2\ 3\ 1\ 3\ 4}$

Start:



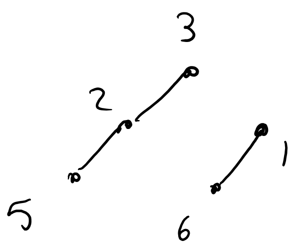
now leafs $\{6,7,2\}$

2 is smallest so $2 — 3$



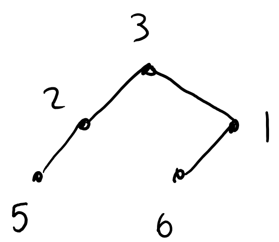
leafs $\{6,7\}$

6 is smallest so $6 \to 1$



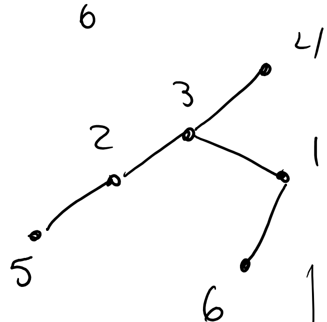
leafs $\{7,1\}$

1 smallest so $1 — 3$



leafs $\{7,3\}$

3 smallest so $3 \to 4$



leafs $\{7,4\}$

so $4 — 7$