Homework 6 Solutions

(1) For the pairs of the form (x,x) you either have them in the relation or not.

For $x \neq y$ you either include both (x,y) and (y,x) or don't include both.

So there are

$$2^{\binom{6}{2}+6} = 2^{21} \text{ symmetric relations.}$$

For reflexive, you must include (a,a), $(b,b), \ldots, (f,f)$. For every $x \neq y$ you can include (x,y) in or not. Therefore there are

$$2^{21-6} = 2^{15} \text{ symmetric relations that are also reflexive.}$$

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(3) It's <u>not reflexive</u> (missing $(5,5)$)

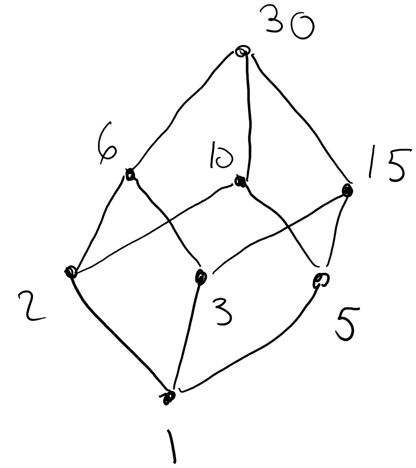
so it's not a poset.

It is antisymmetric

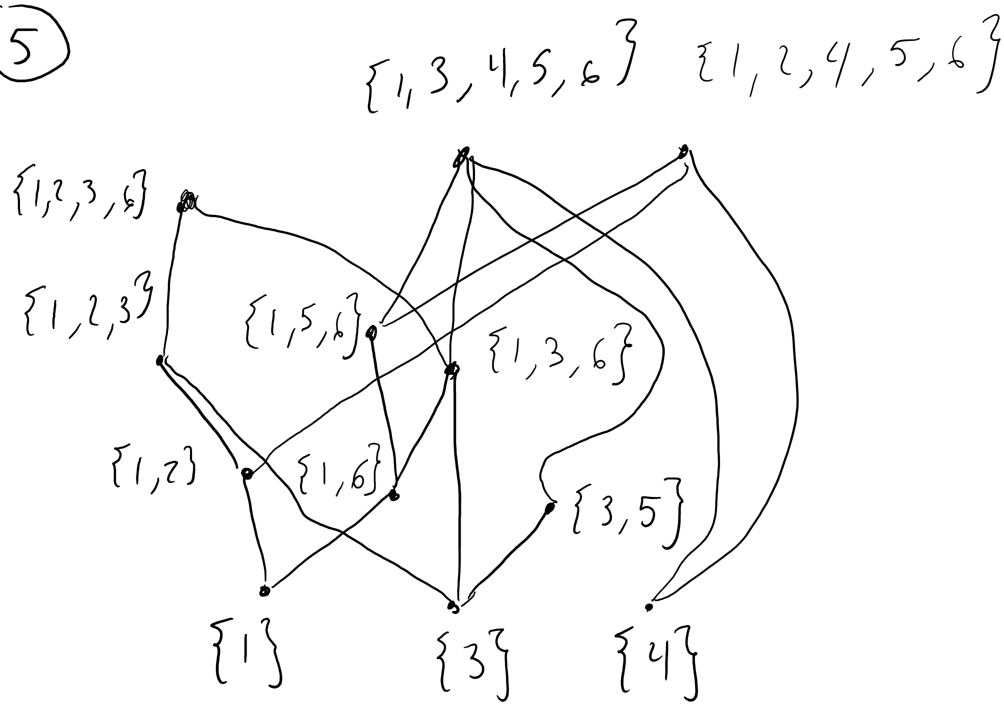
It is transitive.

If we add (5,5) it becomes a poset.

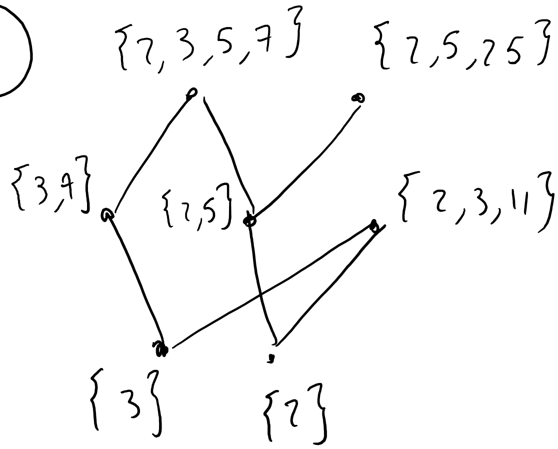
4

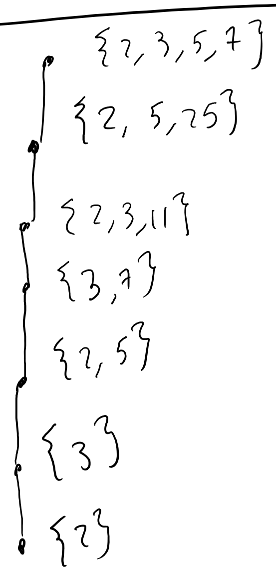


5

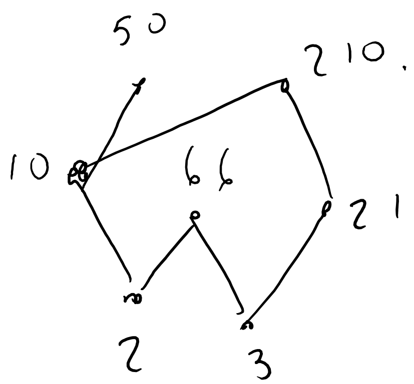


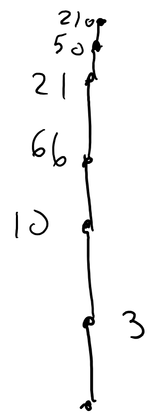
(6)



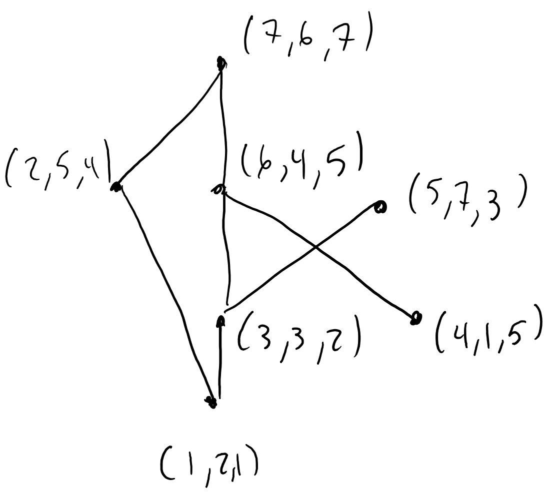


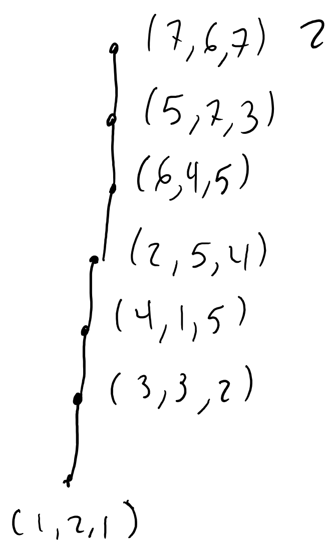
Linear extension





Linear extension



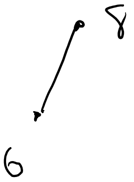


(7) Alice is right, the height is 5, the width is 3.

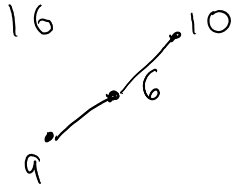
(8) Maximal: 15, 8, 11, 2, 17, 3

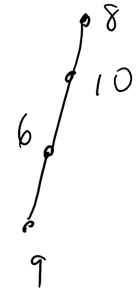
(8) a) maximal: 15, 8, 11, 2, 17, 3

b) minimal: 16, 1, 5, 14

c) 

is the only maximal chain with 2 pts.

d) 

not maximal since 

is longer.

e) $\{16, 1, 5, 14\}$, since this includes all the minimal elements, there are no other elements incomparable to all 4.

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(9) Following the algorithm suggested by the proof of Thm 16.18 we have the following antichains

$\{12, 22, 16, 18, 23\}$
$\{11, 13, 3, 2, 21, 17\}$
$\{10, 4, 25\}$
$\{5, 24, 8\}$
$\{20\}$
$\{19, 9\}$
$\{6, 7\}$
$\{1, 26\}$.

$\{15, 14\}$

Since the algorithm yields h antichains, we have $\underline{h = 9}$.

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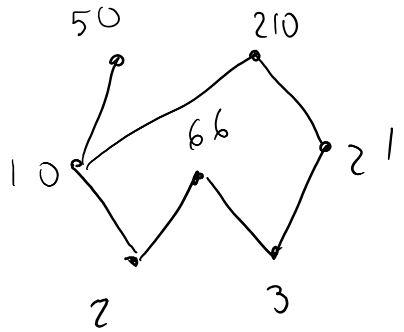
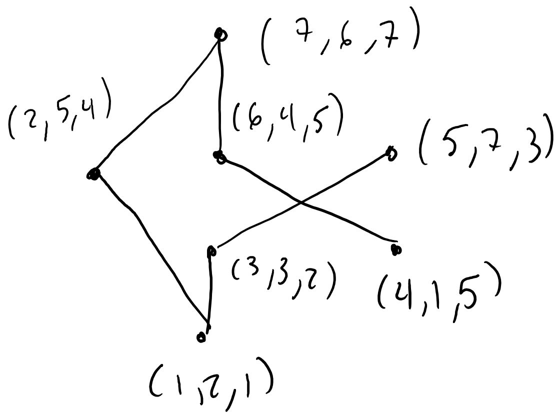
(10)



$\{2, 10, 50\}$
$\{3, 21, 210\}$
$\{66\}$
is a partition into 3 chains, so $w \le 3$.

$\{10, 66, 21\}$ is an antichain, so $w \ge 3$, so $w = 3$.

For the second poset:



$\boxed{\begin{array}{l} \{(1,2,1), (2,5,4), (7,6,7)\} \\ \{(3,3,2), (5,7,3)\} \\ \{(4,1,5), (6,4,5)\} \end{array}}$

partition into 3 chains so $w \le 3$

$\{(2,5,4), (6,4,5), (5,7,3)\}$ is an antichain, so $w \ge 3$, so $w = 3$.

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(11) $\binom{10}{5}$ by Sperner's Theorem.