Understanding Tupper's Self-Referential Formula

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$$\frac{1}{2} < \left\lfloor \mathrm{mod}\left(\left\lfloor \frac{y}{17} \right\rfloor 2^{-17\lfloor x \rfloor - \mathrm{mod}(\lfloor y \rfloor, 17)}, 2 \right) \right\rfloor$$

Background

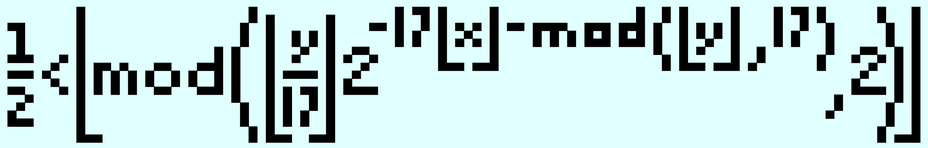
Tupper's Self-Referential Formula:

-The equation tells you which (x, y) coordinates are colored
-At a certain k and k+17 on the y-axis and between 0 and 106 on the x-axis, the plot of the formula is itself

The key is the value of k:

k=48584506361897134235820959624942020445814005879832445494830930850619347047088099284506447698655243648499972470249151191104116057391774078569197543265718554420572104457358836818298237541396343382251994521916512843483329051311931999535024137587652392648746133949068701305622958132194811136853395355652908500238750928568926945559742815463865107300491067230589335860525440966643512653493636439571255656959368151843348576052669401612512669514215505395545191537854575257565907405401579290017659679654800644278291314885482599147212485063526866304763 00

If you plot the equation and look at it between a height of k and k + 17 up the y-axis, it gives the plot of the equation:



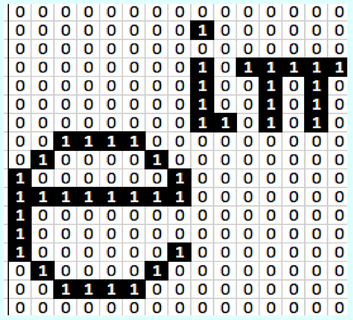
However, Tupper's Self-Referential formula not only plots itself, it plots every 106 × 17 grid of white and black pixels. Therefore, we can find the value of k for anything that will fit in this size plot.

k=144520248970897582847942537337194567481277782215150702479718813968549088735682987348888251320905766438178883231976923440016667764749242125128995265907053708020473915320841631792025549005418004768657201699730466383394901601374319715520996181145249781945019068359500510657804325640801197867556863142280259694206254096081665642417367403940384170774537427319606443899923010379398938675025786929455234476319291860957618345432248004921728033349419816206749854472038193939738513848960476759782673313437697051994580681869819330446336774047268864

k=2352035939949658122140829649197960929306974813625028263292934781954073595495544614140648457342461564887325223455620804204796011434955111022376601635853210476633318991990462192687999109308209472315419713652238185967518731354596984676698288025582563654632501009155760415054499960

Calculating k

→Binary Number
→ Base 10
→ Multiply by 17
→ **k-value** where the plot appears



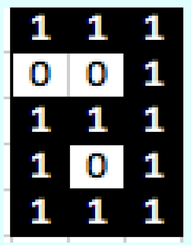
My Project

Write a program to find where the equation plots any three-line phrase in the 106 × 17 area

Making Letters

Formulas for each lowercased letter, each capital letter, a space, a hash tag, an exclamation point, a heart, and a smiley face.

Binary number for the letter a is 11101 10101 11111



Multiply by 2^{17i} to move column to the right

Formula for the lowercased a is:
f["a"] = 17*((1 + 2 + 4 + 16) + (1 + 4 + 16)*2^{17} + (1 + 2 + 4 + 8 + 16)* 2^{34})

Plotting a Word

Sum the characters, allowing 5 spaces per character, to get the formula:

$$\sum_{k=1}^{Length(w)} 2^{17*5*(k-1)} f(w(k))$$

Create function that will calculate the k-value for anything by just defining the word:

```
Speller[word_] := Module[{w = Characters[word]}, Sum[2^(17*5*(k - 1))*f[w[[k]]], {k, 1, Length[w]}]]
```

The function tupperPlotter will plot any k

tupperPlotter[Speller["Margaret"]]:

Plotting Multiple Lines

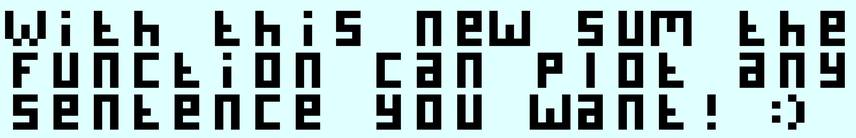
Change the formula for Speller to write multiple lines:

```
Sum[2^{17*5*(k - 1) + 12}*f[w[[k]]],
{k, 1, Min[21, Length[w]]}]
 + If[Length[w] > 21, Sum[2^{17*5*(k - 22) + 6}*f[w[[k]]], {k, 22, Min[42, Length[w]]}], 0] + If[Length[w] > 42,
Sum[2^{17*5*(k - 43)}*f[w[[k]]], {k, 43, Length[w]}], 0]]
```

Now, tupperPlotter[Speller["sentence"]] with this new sum, will plot any sentence that is 63 characters or less

word= "With this new sum the function can plot any sentence you want! :"

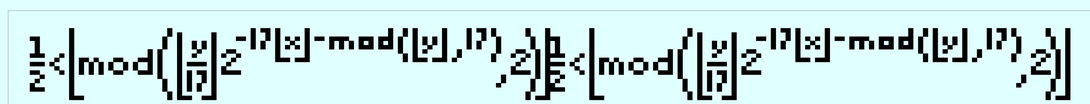
tupperPlotter[word]



Repeating the Formula

The formula will plot for any distance on the x-axis

I also found the k-value that would plot the formula twice (horizontally):



Common Mistake

In computer science positive y often goes downwards, however in mathematics positive y goes upwards.

Therefore, the k-value that most websites say works for self-referencing is actually incorrect.

Bibliography
The 'Everything' Formula. YouTube. Numberphile, 15 Apr. 2015. Web. 30 May 2015.
"How Does Tupper's Self-referential Formula Work?" T he Lumber Room. N.p., 12 Apr. 2011. Web. 30 May 2015.