Report on the 52nd Annual USA Mathematical Olympiad

BÉLA BAJNOK
Gettysburg College
Gettysburg, PA 17325
bbajnok@gettysburg.edu

ENRIQUE TREVIÑO
Lake Forest College
Lake Forest, IL 60045
trevino@lakeforest.edu

The USA Mathematical Olympiad (USAMO) is the final round in the American Mathematics Competitions series for high school students, organized each year by the Mathematical Association of America. The competition follows the style of the International Mathematical Olympiad (IMO): it consists of three problems each on two consecutive days, with an allowed time of four and a half hours both days.

The 52nd annual USAMO was given on Tuesday, March 21, 2023 and Wednesday, March 22, 2023. Of the 238 students taking the exam, 16, 28, and 43 earned Gold, Silver, and Bronze Prizes, respectively; an additional 66 students received Honorable Mention. The names of the prize winners, as well as more information on the American Mathematics Competitions program, can be found on the site https://www.maa.org/math-competitions. Below we present the problems and solutions of the competition; a similar article for the USA Junior Mathematical Olympiad (USAJMO), offered to students in grade 10 or below, can be found in a concurrent issue of the *College Mathematics Journal*.

The problems of the USAMO are chosen—from a large collection of proposals submitted for this purpose—by the USAMO/USAJMO Editorial Board, under the leadership of co-editors-in-chief Oleksandr Rudenko and Enrique Treviño. This year's problems were created by Ankan Bhattacharya, Zack Chroman, Holden Mui, and Carl Schildkraut.

The solutions presented here are those of the present authors, relying in part on the submissions of the problem authors. Each problem was worth 7 points; the nine-tuple

$$(n; a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0)$$

states the number of students who submitted a paper for the relevant problem, followed by the numbers who scored $7, 6, \ldots, 0$ points, respectively.

Problem 1 $(226; 171, 2, 2, 1, 2, 0, 10, 38)$; *proposed by Holden Mui.* In an acute triangle ABC, let M be the midpoint of $\overline{BC}$. Let P be the foot of the perpendicular from C to AM. Suppose that the circumcircle of triangle ABP intersects line BC at two distinct points B and Q. Let N be the midpoint of $\overline{AQ}$. Prove that $NB = NC$.

First solution. We will prove that N lies on the perpendicular bisector of BC, from which the claim follows. Let T be the intersection point of AQ with the perpendicular bisector of BC; we need to prove that $T = N$.

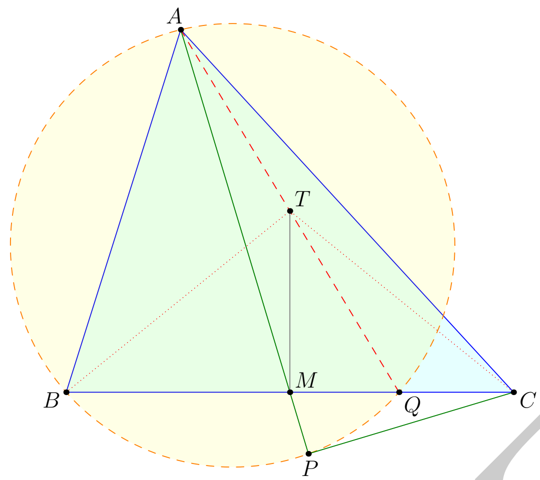

Figure 1: Illustration for the first solution to Problem 1.

Applying the Law of Sines to triangle ATM, we have

$$AT = AM \cdot \frac{\sin(\angle AMT)}{\sin(\angle ATM)}. \tag{1}$$

Now

$$\angle AMT = 90^\circ - \angle BMA = 90^\circ - \angle CMP = \angle PCM,$$

so

$$\sin(\angle AMT) = \sin(\angle PCM) = \frac{PM}{MC} = \frac{PM}{BM}.$$

Furthermore, $\angle ATM$ and $\angle MTQ$ are supplementary angles, so

$$\sin(\angle ATM) = \sin(\angle MTQ) = \frac{MQ}{TQ}.$$

Therefore, we may rewrite (1) as

$$AT = \frac{AM \cdot PM \cdot TQ}{BM \cdot MQ}.$$

Since $\angle QAM = \angle QAP = \angle QBP = \angle MBP$ and $\angle AMQ = \angle PMB$, triangles AMQ and BMP are similar, and thus $AM/MQ = BM/PM$. This gives $AT = TQ$, as claimed.

Second solution. We let ω_1 be the circumcircle of triangle ABP, and ω_2 be the circumcircle of triangle APC. The foot of the perpendicular from A to BC, denoted by D, is then on ω_2, since $\angle ADC = \angle APC$.

From the Power of a Point Theorem applied to circles ω_1 and ω_2, respectively, we get

$$MA \cdot MP = MB \cdot MQ$$

and

$$MA \cdot MP = MC \cdot MD;$$

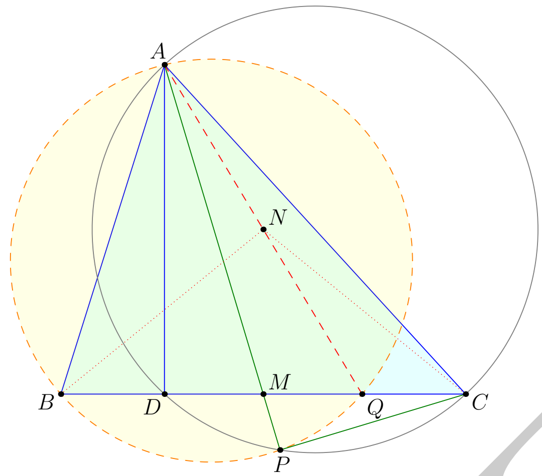

Figure 2: Illustration for the second solution to Problem 1.

since $MB = MC$, we get $MD = MQ$. But then MN is a midline of triangle ADQ, and thus MN and AD are parallel. Therefore, N lies on the perpendicular bisector of BC, which implies our claim.

Problem 2 $(211; 55, 2, 7, 0, 8, 24, 64, 51)$; *proposed by Carl Schildkraut.* Let $\mathbb{R}^+$ be the set of positive real numbers. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that, for all $x, y \in \mathbb{R}^+$, we have

$$f(xy + f(x)) = xf(y) + 2. \tag{2}$$

Solution. We claim that the only such function is $f(x) = x+1$. It is easy to verify that this function works; we need to prove that there are no others.

We start by showing that f is injective. Suppose that x_1 and x_2 are positive real numbers. Applying the given equation to $x = x_1$ and $y = x_2$ results in

$$f(x_1x_2 + f(x_1)) = x_1f(x_2) + 2,$$

and for $x = x_2$ and $y = x_1$ we get

$$f(x_1x_2 + f(x_2)) = x_2f(x_1) + 2.$$

From this it follows that $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Suppose now that u and v are arbitrary positive real numbers. Letting $x = f(u)$ and $y = v$, (2) becomes

$$f(vf(u) + f(f(u))) = f(u)f(v) + 2,$$

and letting $x = f(v)$ and $y = u$ gives

$$f(uf(v) + f(f(v))) = f(v)f(u) + 2.$$

Since the two right sides are equal and f is injective, we find that

$$uf(v) + f(f(v)) = vf(u) + f(f(u)) \tag{3}$$

holds for all $u, v \in \mathbb{R}^+$.

Next, we let $u = x$, $v = 1$, and $f(1) = a$; (3) then yields

$$ax + f(a) = f(x) + f(f(x)).$$

Similarly, with $u = x$, $v = 2$, and $f(2) = b$, we get

$$bx + f(b) = 2f(x) + f(f(x)).$$

Adding the last equation to the negative of the one before it, we arrive at

$$f(x) = (b - a)x + f(b) - f(a). \tag{4}$$

Therefore,

$$f(a) = (b - a)a + f(b) - f(a)$$

and

$$f(b) = (b - a)b + f(b) - f(a);$$

subtraction yields

$$f(b) - f(a) = (b - a)^2.$$

Let us write $c = b - a$; with that, (4) becomes $f(x) = cx + c^2$. To determine c, we plug in $x = y = 1$ in (2):

$$f(1 + f(1)) = f(1) + 2,$$

which gives

$$c(1 + c + c^2) + c^2 = c + c^2 + 2,$$

and thus

$$c^3 + c^2 - 2 = (c - 1)(c^2 + 2c + 2) = 0.$$

The only real number solution of this equation is $c = 1$. Therefore, $f(x) = x + 1$, completing our proof.

Problem 3 $(186; 24, 17, 10, 9, 9, 12, 40, 65)$; *proposed by Holden Mui.* Consider an n-by-n board of unit squares for some odd positive integer n. We say that a collection C of identical dominoes is a *maximal grid-aligned configuration* on the board if C consists of $(n^2 - 1)/2$ dominoes where each domino covers exactly two neighboring squares and the dominoes do not overlap: C then covers all but one square on the board. We are allowed to slide (but not rotate) a domino on the board to cover the uncovered square, resulting in a new maximal grid-aligned configuration with another square uncovered. Let $k(C)$ be the number of distinct maximal grid-aligned configurations obtainable from C by repeatedly sliding dominoes. Find all possible values of $k(C)$ as a function of n.

Solution. The answer is that $k(C)$ may be any positive integer up to and including $(n - 1)^2/4$, as well as $(n + 1)^2/4$. Below we assume that $n \geq 3$.

We first prove that $k(C)$ is at most $(n + 1)^2/4$. We label the squares of the board by ordered pairs (i, j) in the usual manner, and then color each square by one of three colors: if both of its coordinates are odd, we color it *red*; if both of its coordinates are even, we color it *blue*; and if its coordinates have different parities, we color it *white*. Note that we then have exactly $(n+1)^2/4$ red squares, $(n-1)^2/4$ blue squares (together, $(n^2+1)/2$ *dark* squares), and $(n^2-1)/2$ white squares.

In a maximal grid-aligned configuration, each domino covers a dark square and a white square. We will refer to each domino by the color of the dark square it covers. We will call the square that is not covered by dominoes *empty*. Note that, by parity, the empty square is colored dark.

Let Γ be the directed graph whose vertices are the dark squares, and whose directed edges are drawn from a square v to the square that the domino covering v points to (the square that, if it were empty, the domino could slide to in one move), if it exists. An example is shown in Figure 3. Note that Γ uniquely determines the configuration of dominoes.

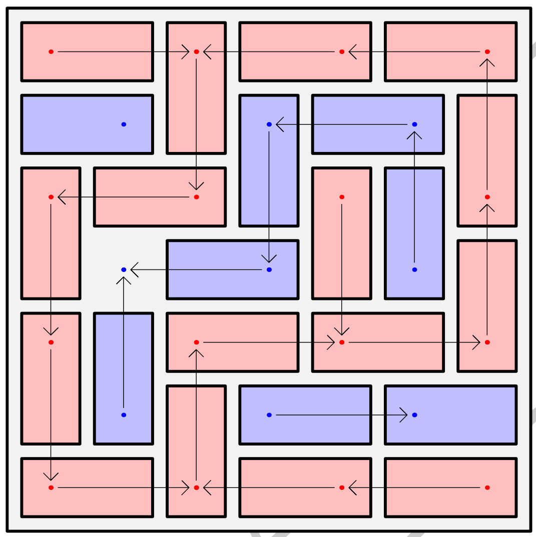

Figure 3: Illustration for a maximal grid-aligned configuration and its corresponding directed graph.

Let G be the undirected graph corresponding to Γ. Observe that the connected components of G are formed by monochromatic dominoes (i.e., either all red or all blue). Suppose that C is a cycle in G. Connecting the center points of the unit squares in C forms a polygon P. Since each side of P has even length, the region surrounded by P can be divided into 2×2 squares, so the area of the region is divisible by 4, and is thus even. By Pick's Theorem, the area equals $B/2+I-1$, where B is the number of unit squares in C, and I is the number of unit squares in the interior of P. Since B is twice the number of dominoes in C, and C contains an even number of dominoes, we find that I is odd, which can only be true if C encloses the empty square (i.e., contains it in its interior).

Let T be the connected component of G that contains the empty square u; according to what we just proved, T cannot contain a cycle, so it must be a tree. Let $\Gamma(T)$ be the subgraph of Γ on T. Since T is a tree, it is valid to say whether the edges in $\Gamma(T)$ point towards u or away from it. We claim that, in fact, each directed edge in $\Gamma(T)$ points towards u. Suppose indirectly that there are some directed edges that point away from u, and choose one whose tail vertex v_1 is closest to u; let v_2 be the head of this directed edge. Let v_3 be the vertex in T that v_1 is adjacent to along the path from v_1 to u. By assumption, the edge connecting v_1 and v_3 has v_1 as its tail. But v_2 and v_3 are different vertices, so v_1 is the tail of more than one directed edge, which is a contradiction.

This means that u is the universal sink of $\Gamma(T)$. We thus find that there is exactly one way to make any vertex of $\Gamma(T)$ become the empty square: reverse the direction of each edge on the path that connects that vertex to u, that is, slide each corresponding domino on that path towards u. Therefore, $k(C)$ equals the number of vertices in T. Since T is monochromatic, it can have at most as many vertices as there are blue squares or red squares, which implies that $k(C)$ is at most $(n+1)^2/4$. We can achieve $k(C) = (n+1)^2/4$ by positioning the dominoes covering red squares in a snake-like fashion. An example construction for $n = 7$ is shown in Figure 4.

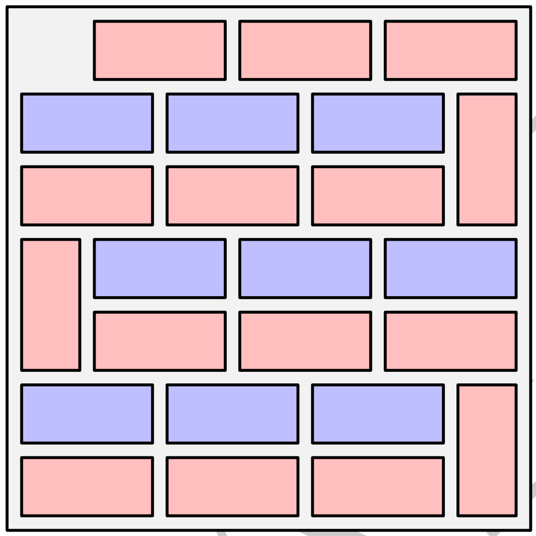

Figure 4: A maximal grid-aligned configuration achieving $k(C) = (n+1)^2/4$.

Suppose now that $k(C)$ is more than $(n-1)^2/4$; we will prove that $k(C)$ then equals $(n+1)^2/4$. Let T be the connected component in G containing u. Since T is monochromatic and has more than $(n-1)^2/4$ vertices, u must be red. We will prove that every vertex in T is a red square.

Since there are only $(n-3)^2/4$ red squares in the interior of the board, T must contain a square on the boundary. As we may slide dominoes within T, without loss of generality we may assume that u is on the boundary of the board. Since each red vertex other than u is the tail of an edge in Γ, if v were not in U, then v would be on a cycle, but that is impossible as such a cycle would need to contain u in its interior, but u is on the boundary of the board. This proves that every red vertex of G is connected to u and thus $k(C)$ equals the number of red squares, as claimed.

It remains to be shown that each value in $\{1, 2, \ldots, (n-1)^2/4\}$ may equal $k(C)$ for some maximal grid-aligned configuration C. One possible construction involves positioning the dominoes covering blue cells in a snake-like fashion, blocking the snake's path with a red domino and an empty square, and filling the rest of the grid with red dominoes. An example construction for $n = 7$ and $k(C) = 5$ is shown in Figure 5.

Problem 4 $(226; 143, 6, 6, 3, 4, 5, 31, 28)$; *proposed by Carl Schildkraut.* A positive integer a is selected, and some positive integers are written on a board. Alice and Bob play the following game. On Alice's turn, she must replace some integer n on the board with $n + a$, and on Bob's turn, he must replace some even integer n on the board with $n/2$. Alice goes first and they alternate turns. If on his turn Bob has no valid moves, the game ends.

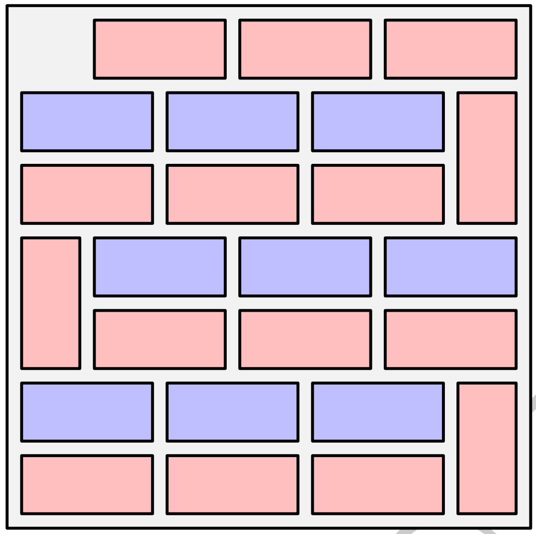

Figure 5: A maximal grid-aligned configuration with $n = 7$ and $k(C) = 5$.

After analyzing the integers on the board, Bob realizes that, regardless of what moves Alice makes, he will be able to force the game to end eventually. Show that, in fact, for this value of a and these integers on the board, the game is guaranteed to end regardless of Alice's or Bob's moves.

Solution. If there is only a single integer on the board, then all moves are determined uniquely, and thus there is nothing to prove. Below we suppose that there are at least two numbers on the board.

Before we make a precise claim regarding the game, let us recall that every positive integer m can be written uniquely as $m = 2^k \cdot c$ where c is odd; k is called the 2-*adic valuation* of m and is denoted by $\nu_2(m)$. Here, for our fixed value of a, we say that a positive integer m is 2-*perfect* if $\nu_2(m) = \nu_2(a)$, 2-*abundant* if $\nu_2(m) > \nu_2(a)$, and 2-*deficient* if $\nu_2(m) < \nu_2(a)$. We can now make the following claims: If the board has at least one number that is 2-perfect or 2-abundant, then Alice can prolong the game indefinitely, but if all numbers on the board are 2-deficient, then the game will terminate (in a finite number of steps) no matter how Alice and Bob play.

For our proof, it is helpful to make the following observations about any positive integer m:

- if m is 2-perfect, then $\nu_2(m + a) > \nu_2(m)$, and thus $m + a$ is 2-abundant;
- if m is 2-abundant, then $\nu_2(m + a) = \nu_2(a)$, and thus $m + a$ is 2-perfect;
- if m is 2-deficient, then $\nu_2(m + a) = \nu_2(m)$, and thus $m + a$ remains 2-deficient.

To verify the first observation, let $m = 2^k \cdot c_1$ and $a = 2^k \cdot c_2$ for some nonnegative integer k and odd integers c_1 and c_2. Then

$$m + a = 2^k \cdot (c_1 + c_2),$$

where $c_1 + c_2$ is even and hence $\nu_2(m+a) > k$. The other two observations can be deduced similarly.

Now we turn to the proof of our claims. Assume first that the board contains at least one number that is 2-perfect or 2-abundant; we claim that Alice can then move in such a way that after her move the board contains at least one 2-abundant number. Indeed, if there is already a 2-abundant number on the board, she should leave it unchanged and add a to any other number, and if there is a 2-perfect one, she can add a to it to make it 2-abundant. (Note that Alice may have several such options, in which case she can choose any of them.) After Alice's move, Bob will have some moves available (for example, on the 2-abundant number on the board). He will reduce the 2-adic valuation of one of the numbers on the board by exactly 1, but that will transfer the scenario back to a case in which the board contains at least one number that is 2-perfect or 2-abundant. Alice can thus use this strategy to prolong the game indefinitely.

Suppose now that all numbers on the board are 2-deficient. We let $S = \sum \nu_2(n)$, where the summation is taken over all integers on the board. By our observations above, every move that Alice makes will leave S unchanged, while obviously each move that Bob makes reduces S by 1. But then, after a finite number of rounds, there will be no more even numbers on the board, and the game terminates.

Problem 5 $(194; 98, 10, 13, 6, 2, 9, 16, 40)$; *proposed by Ankan Bhattacharya.* Let $n \geq 3$ be an integer. We say that an arrangement of the numbers $1, 2, \ldots, n^2$ in an $n \times n$ table is *row-valid* if the numbers in each row can be permuted to form an arithmetic progression, and *column-valid* if the numbers in each column can be permuted to form an arithmetic progression. For what values of n is it possible to transform any row-valid arrangement into a column-valid arrangement by permuting the numbers in each row?

Solution. The answer is that the transformation is always possible when n is prime, but not always when n is composite.

Suppose first that n is prime, and let C be any $n \times n$ row-valid table. Observe that, for each remainder mod n, there are exactly n positive integers up to n^2 that leave that remainder mod n. Let us call a row of C *uniform* if its n elements all leave the same remainder mod n, and *diverse* if the n elements all leave a different remainder mod n.

It is easy to see that each row of C is either uniform or diverse. Indeed, if we had two integers in a row that left the same remainder mod n, then their difference, which is a multiple of the common difference of the arithmetic progression corresponding to that row, would be a multiple of n, and thus all elements in that row would leave the same remainder mod n. Observe also that if C contains a uniform row, then that row includes all numbers with that remainder, so C cannot also contain a diverse row.

Therefore, we have two cases for our row-valid table: either every row is uniform or every row is diverse. In the first case, we can permute the elements in each row so that the jth column contains the elements of the arithmetic progression

$$1 + (j-1)n,\ 2 + (j-1)n,\ \ldots,\ n + (j-1)n$$

in some order, while in the second case, we can arrange that the jth column contains the arithmetic progression

$$j,\ j+n,\ j+2n,\ \ldots,\ j+(n-1)n$$

in some order. This proves our claim when n is prime.

Suppose now that n is composite with a proper divisor $c > 1$. To construct an example for a row-valid table A that cannot be transformed into a column-valid one, we arrange the first n^2

positive integers by setting the jth entry in row i to be

$$a_{i,j} = \begin{cases} i + (j-1)c & \text{if } i \le c; \\ j + (i-1)n & \text{if } i > c. \end{cases}$$

(Thus the first cn positive integers occupy the first c rows of A, arranged in order by columns, and the rest of the integers are placed in the last $n-c$ rows, arranged in order by rows. An illustration for $n = 9$ and $c = 3$ is provided below.)

$$\begin{bmatrix} 1 & 4 & 7 & 10 & 13 & 16 & 19 & 22 & 25 \\ 2 & 5 & 8 & 11 & 14 & 17 & 20 & 23 & 26 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \\ 28 & 29 & 30 & 31 & 32 & 33 & 34 & 35 & 36 \\ 37 & 38 & 39 & 40 & 41 & 42 & 43 & 44 & 45 \\ 46 & 47 & 48 & 49 & 50 & 51 & 52 & 53 & 54 \\ 55 & 56 & 57 & 58 & 59 & 60 & 61 & 62 & 63 \\ 64 & 65 & 66 & 67 & 68 & 69 & 70 & 71 & 72 \\ 73 & 74 & 75 & 76 & 77 & 78 & 79 & 80 & 81 \end{bmatrix}$$

Clearly, each row in A is an arithmetic progression. Suppose, indirectly, that it is possible to permute the elements in the rows so that the table becomes column-valid; let table B be the result of these permutations. Consider the column of B that contains 2. The elements in that column cannot contain 1, since the arithmetic progression then would be $(1, 2, \ldots, n)$, which includes $2 + c$ (since $2 + c \le n$ for $n \ge 4$), but 2 and $2 + c$ are both in the second row (since $2 + c = a_{2,2}$).

Therefore, the elements in B in the column containing 2 are $\{2 + kd \mid k = 0, 1, \ldots, n-1\}$ for some positive integer d. The largest element in this set is $2 + (n-1)d$, for which we must have

$$n^2 - n + 1 \le 2 + (n-1)d \le n^2$$

as this element is in the last row of A; and these inequalities yield $d = n$. We have arrived at a contradiction, since the integer $2 + n$ is then in the column containing 2 (with $k = 1$), but $2 + n$ is in the same row as 2 (since $a_{2,j} = 2 + n$ for $j = n/c + 1$). This is impossible.

Problem 6 $(166; 22, 1, 0, 1, 0, 1, 0, 141)$; *proposed by Zack Chroman.* Let ABC be a triangle with incenter I and excenters I_a, I_b, I_c opposite A, B, and C, respectively. Given an arbitrary point D on the circumcircle of $\triangle ABC$ that does not lie on any of the lines IIa, I_bI_c, or BC, suppose the circumcircles of $\triangle DIIa$ and $\triangle DI_bI_c$ intersect at two distinct points D and F. Let E be the intersection of lines DF and BC. Prove that $\angle BAD = \angle EAC$.

Solution. Let ω, ω_1, and ω_2 be the circumcircles of $\triangle ABC$, $\triangle DII_a$, and $\triangle DI_bI_c$, respectively. Let P be the intersection of ω and ω_1 that is not D.

Since I is the incenter and I_a is an excenter, $\angle ICI_a = \angle IBI_a = 90°$, so $BICI_a$ is cyclic; let us call this circle ω_a. Since PD is the radical axis of ω and ω_1, BC is the radical axis of ω_a and ω, and II_a is the radical axis of ω_a and ω_1, we have that PD, BC and II_a concur at a point K.

Let M be the intersection of II_a and ω. Since AI is an angle bisector of $\angle BAC$, M lies in the midpoint of the arc $\widehat{BC}$ in ω. Let E' be the intersection of PM with BC. Let $\angle BAM = \alpha, \angle ABC = \beta$, and note that because M is on the angle bisector of $\angle BAC$, $\angle MBC = \angle MAC = \angle BAM = \alpha$. Then $\angle ABM = \alpha + \beta$, while $\angle BKM = 180° - \angle BKA = 180° - (180° - (\alpha + \beta)) = \alpha + \beta$.

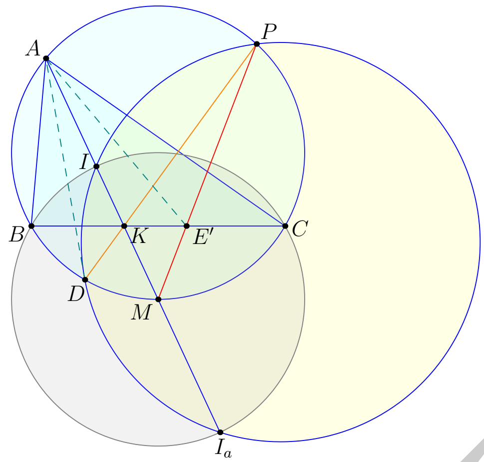

Figure 6: Figure focusing on the interaction between ω and ω_1.

By the Law of Sines applied to triangles ABM and BKM, we have

$$\frac{MB}{\sin(\alpha)} = \frac{MA}{\sin(\alpha+\beta)}$$

and

$$\frac{MB}{\sin(\alpha+\beta)} = \frac{MK}{\sin(\alpha)},$$

and therefore

$$MK \cdot MA = MB^2.$$

Similarly, we can show that

$$ME' \cdot MP = MB^2. \tag{5}$$

But then

$$ME' \cdot MP = MK \cdot MA,$$

which implies, by the Power of a Point Theorem, that $AKE'P$ is a cyclic quadrilateral.

Then

$$\angle KAE' = \angle KPE' = \angle DPM = \angle DAM,$$

and thus

$$\angle BAD = \angle BAM - \angle DAM = \angle CAM - \angle E'AM = \angle E'AC.$$

It follows that $\angle BAD = \angle E'AC$.

Now let Q be the intersection of ω and ω_2 that is not D. Note that, because exterior angle bisectors and interior angle bisectors are perpendicular, $\angle I_bBI_c = \angle I_bCI_c = 90°$, so BCI_cI_b is a cyclic quadrilateral. Let the circle containing BCI_cI_b be called ω_{bc}. Since QD is the radical axis of ω and ω_2, BC is the radical axis of ω_{bc} and ω, and I_bI_c is the radical axis of ω_{bc} and ω_2, we have that QD, BC, and I_bI_c concur at a point L.

Let N be the midpoint of the arc $\widehat{BC}$ but on the other side of M, that is, N is the intersection of I_bI_c and ω other than A. Let E'' be the intersection of QN and BC. By analogous reasoning

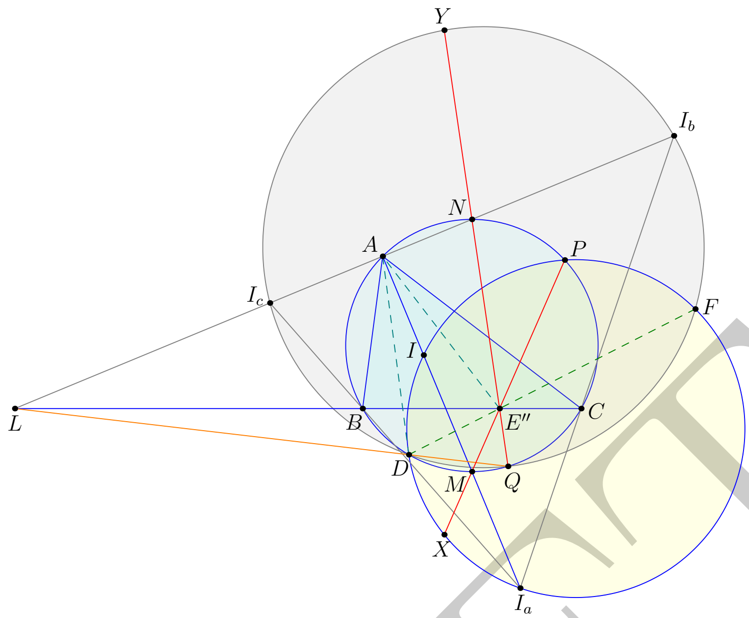

Figure 7: Figure once we include the interactions with ω_2.

as above, we can show that $ALE''Q$ is cyclic and get that $\angle BAD = \angle E''AC$. Since E'' is on the segment BC and satisfies $\angle E''AC = \angle E'AC$, we must have $E' = E''$. Therefore $PM \cap QN = E'$.

Let X and Y be the reflections of E' over M and N, respectively. Note that

$$\angle IBM = \angle IBC + \angle CBM = \angle IBA + \angle IAC = \angle IBA + \angle IAB = \angle BIM,$$

and thus $MI = MB$. We can similarly show $MB = MI_a$, and conclude that $MB = MI = MI_a$. Using this and (5) yields

$$MX \cdot MP = ME' \cdot MP = MB^2 = MI \cdot MI_a,$$

proving that X lies on ω_1. Similarly, we see that Y lies on ω_2. Now since $E' = \overline{PM} \cap \overline{QN}$, we have $E'P \cdot E'M = E'Q \cdot E'N$, so

$$E'P \cdot E'X = E'P \cdot (2E'M) = E'Q \cdot (2E'N) = E'Q \cdot E'Y,$$

proving that E' has equal powers in ω_1 and ω_2. Therefore, E' lies on their radical axis DF. Since E' is on DF and on BC, it must be that $E' = E$. We can now conclude that $\angle BAD = \angle EAC$.

Acknowledgments. The authors wish to thank Chris Jeuell for a careful reading of this article. Credit for Figures 1, 2, 6, and 7 goes to Evan Chen and for Figures 3, 4, and 5 to Holden Mui.

Summary. We present the problems and solutions to the 52nd Annual United States of America Mathematical Olympiad.

BÉLA BAJNOK (MR Author ID: 314851) is a professor of mathematics at Gettysburg College and the director of the American Mathematics Competitions program of the MAA.

ENRIQUE TREVIÑO (MR Author ID: 894315) is an associate professor of mathematics at Lake Forest College and the co-editor-in-chief of the USA(J)MO Editorial Board.