Sample Exam I Questions

1. Let \( \mathbf{a} = \langle 3, -2, 7 \rangle \) and \( \mathbf{b} = \langle -1, 1, 1 \rangle \). Calculate:
   (a) \( \mathbf{a} + \mathbf{b} \)   (b) \( \|\mathbf{a}\| + \|\mathbf{b}\| \)  
   (c) \( \mathbf{a} \cdot \mathbf{b} \)   (d) \( \mathbf{a} \times \mathbf{b} \)

2. Let \( \mathbf{a} = \langle 2, 0, 1 \rangle \) and \( \mathbf{b} = \langle -3, 1, 0 \rangle \).
   (a) Calculate \( \mathbf{a} - 3\mathbf{b} \).
   (b) Find a unit vector in the direction of \( \mathbf{a} \).
   (c) Calculate the angle between \( \mathbf{a} \) and \( \mathbf{b} \) to the nearest degree.
   (d) Calculate the scalar component of \( \mathbf{a} \) onto \( \mathbf{b} \). (scalar projection)

3. Let \( \mathbf{a} = 3\mathbf{i} - 4\mathbf{j} - \mathbf{k} \) and \( \mathbf{b} = \mathbf{i} + 3\mathbf{k} \).
   (a) Calculate \( -\mathbf{a} + 2\mathbf{b} \cdot \mathbf{b} \).
   (b) Calculate the angle between \( \mathbf{a} \) and \( \mathbf{b} \).
   (c) Find a unit vector perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \).

4. Find the area of the triangle formed by the vertices \((1, -1, 0), (3, 4, -1)\), and \((-1, -1, 2)\).

5. Find the volume of the parallelepiped determined by the three vectors \( \mathbf{a} = \langle 2, 3, -2 \rangle \), \( \mathbf{b} = \langle 1, -1, 0 \rangle \), and \( \mathbf{c} = \langle 0, 2, 3 \rangle \).

6. Find an equation of the line through the point \((1, 2, 3)\) in the direction of \(\mathbf{i} - \mathbf{k}\).

7. Find an equation of the line perpendicular to the plane \(3x - 2z = 1\) going through the point \((1, 2, 3)\).

8. Find an equation of the plane perpendicular to the line \(x = t - 1, y = -t - 3, z = 7\) and passing through the point \((1, 2, 3)\).

9. Find an equation of the plane containing both the lines \(x = t - 1, y = -t + 4, z = 3\) and \(x = 3t + 1, y = t + 2, z = -t + 3\).

10. Find an equation of the line that is perpendicular to both the lines in problem 9 above and passes through the point \((1, 2, 3)\).

11. Find an equation of the line that passes through the point \((1, 2, 3)\) and is parallel to both the planes \(x - y = 0\) and \(3x + y - z = 2\).

12. Find an equation of the plane through the point \((1, 2, 3)\) and is parallel to the line \(x = -t, y = t + 1, z = 2\) and perpendicular to the plane \(3x + y - z = 2\).

13. Find an equation of the plane through the three points \((1, 2, 3), (4, 5, 6),\) and \((-2, 0, 4)\).

14. Given the following curves, find the position, velocity, speed, and acceleration at \(t = 1\).
   (a) \( \mathbf{r}(t) = \cos(t)\mathbf{i} + (2 + \sin(t))\mathbf{j} + (4t + 3)\mathbf{k} \)   (b) \( \mathbf{r}(t) = (t^2 - 3, t^3 - 6, 7 - t^2) \)

15. Given the curves defined in #14 above, find the tangent line to the point \((1, 2, 3)\).

16. For each curve in #14 above, find the arclength from \(t = 0\) to \(t = 1\).

17. Given an object that accelerates according to the following acceleration function, with given initial conditions, find the position of the object at time \(t = 2\).
   (a) \( \mathbf{a}(t) = \langle 12t, -12t^2 \rangle \), \( \mathbf{r}(0) = \langle 1, 2 \rangle \), \( \mathbf{v}(0) = \langle 0, 6 \rangle \)
   (b) \( \mathbf{a}(t) = \langle e^{t/2}, 0, 6 \rangle \), \( \mathbf{r}(0) = \langle 1, 2, 3 \rangle \), \( \mathbf{v}(0) = \langle 0, 6, 0 \rangle \)

18. The graph of \(z = xy^2\) matches choice...