Solutions to Sample Exam 2 Questions

1. (a) \( \frac{\partial z}{\partial x} = \ln y \), \( \frac{\partial z}{\partial y} = \frac{x}{y} + 2y \) (b) \( \frac{\partial z}{\partial x} = (1 + y)e^{x+xy} \), \( \frac{\partial z}{\partial y} = xe^{x+xy} \) (c) \( \frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2 - y^2}} \), \( \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{x^2 - y^2}} \)

2. (a) \( \frac{\partial w}{\partial x} = y, \frac{\partial w}{\partial y} = x + z^2, \frac{\partial w}{\partial z} = 2yz \).

3. (a) \( Df = [1 \quad -z \quad -y] \) (b) \( DF = \begin{bmatrix} y & x \\ 2x & 0 \end{bmatrix}, \frac{\partial^2 z}{\partial x \partial y} = 1 \) (b) \( \frac{\partial^2 z}{\partial x \partial y} = y(y-1)x^{y-2}, \frac{\partial^2 z}{\partial x \partial y} = x^{y-1} + yx^{y-1} \ln x \).

4. (a) \( f_{xyz} = 27x^2y^2z^2, f_{xxz} = 18xy^3z^2 \) (b) \( f_{xyz} = 60e^{3x+4y+5z}, f_{xxz} = 45e^{3x+4y+5z} \).

5. Yes, you should get \(-13e^{-13t}\) \sin 2x \cos 3y for both sides after differentiating.

6. \( Df = \begin{bmatrix} 1 & 2y & 3z^2 \end{bmatrix} \frac{\partial^2 w}{\partial x \partial y} = 0, \frac{\partial^2 w}{\partial x \partial y} = 1 \) (b) \( \frac{\partial^2 w}{\partial x \partial y} = y(y-1)x^{y-2}, \frac{\partial^2 w}{\partial x \partial y} = x^{y-1} + yx^{y-1} \ln x \).

7. \( \frac{\partial w}{\partial x} = yx^{y-1} + 2x^2tx^y \ln x, \frac{\partial w}{\partial y} = yx^{y-1} + 2x^2x^y \ln x \).

8. \( \frac{\partial u}{\partial x} = zy^2e^{xz} + 2ye^{xz}(\frac{1}{x} + 5x^4) + xy^2e^{xz}2x \cos x^2 \).

9. (a) \( 1 \) (b) \( \frac{1}{2} \) (c) \( \frac{1}{2} \) (d) \( \frac{1}{2} \) (e) \( \frac{1}{2} \) (f) \( \frac{1}{2} \)

10. 11. \( \frac{\partial z}{\partial t} = -10, \frac{\partial z}{\partial \theta} = -22 \).

11. \( \frac{\partial w}{\partial s} = -10, \frac{\partial z}{\partial t} = -22 \).

12. \( \frac{\partial w}{\partial s} = -10, \frac{\partial z}{\partial t} = -22 \).

13. \( \frac{\partial w}{\partial s} = -10, \frac{\partial z}{\partial t} = -22 \).

14. \( \frac{\partial w}{\partial s} = -10, \frac{\partial z}{\partial t} = -22 \).

15. (a) \( z = 4y^2 + 5 \) (b) \( 2x - 4y - z + 3 = 0 \) (c) \( x - y + 2z = 0 \)

16. (a) \( -\frac{10\pi}{3} \) (b) \( \frac{\theta}{2}, \frac{\tau}{2} \)

17. (a) \( -\frac{10\pi}{3} \) (b) \( \frac{\theta}{2}, \frac{\tau}{2} \)

18. (9,2)

19. (a) Critical points at \((0,0)\) \( f = 0 \) and \((\pm \frac{1}{2\sqrt{2}}, -\frac{1}{4})\) \( f = -\frac{1}{128} \). No global max. global mins at \((\pm \frac{1}{2\sqrt{2}}, -\frac{1}{4})\). (b) Critical points at \((0, -\frac{1}{\sqrt{2}})\) \( f \approx -42888 \) and \((0, \frac{1}{\sqrt{2}})\) \( f \approx 42888 \) and \((\frac{1}{\sqrt{2}}, \frac{1}{2})\) \( f \approx 47237 \). Global max at \((\pm \frac{1}{\sqrt{2}}, \frac{1}{2})\), global min at \((0, -\frac{1}{\sqrt{2}})\).

20. \( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \) (a) \( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \) (b) \( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \)

21. \( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \) (a) \( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \) (b) \( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \)

22. (a) The system \( 2x = 2\lambda, 8y = 3\lambda, 2x + 3y = 25 \) has only one solution \((x, y) = (8,3)\), so \( f(8,3) = 100 \) must be the min. (b) The system \( 2x + \lambda, 3 = 8y + x^2 + 4y^2 = 4 \) has two solutions \((x, y) = (-8/5, -3/5)(f = -5)\) and \((x, y) = (8/5, 3/5)(f = 5)\). So \( f = -5 \) must be min and \( f = 5 \) must be max. (c) The system \( y = 2x\lambda, 2x + 2y = 2y\lambda, x^2 + y^2 = 24 \) has four solutions \((3, -\sqrt{15})(f = -5\sqrt{15} \approx -19.3649)\) \( [MIN] \), \((3, \sqrt{15})(f = 5\sqrt{15} \approx 19.3649)\) \( [MAX] \), \((-4, -2\sqrt{2})(f = 4\sqrt{2} \approx 5.6568)\), \((-4, 2\sqrt{2})(f = -4\sqrt{2} \approx -5.6568)\). (d) The system is \( y = 2x\lambda, x = 4y\lambda, 2z = 6z\lambda, x^2 + 2y^2 + 3z^2 = 1 \).

23. (a) Local max at \((4,4)\)

(b) Local min at \((0,0)\), local max at \((-\frac{\sqrt{2}}{2}, 0)\), saddle points at \((-1, \pm 2)\).

(c) Saddle point at \((0,0)\).

(d) Local max at \((1, -1)\), saddle point at \((1, 1)\).

(e) No critical points

(f) Saddle point at \((\frac{\sqrt{2}}{2}, \frac{3}{2})\).