Practice Exam 3

Solutions

1)

a) Using Table A-2 we find that the probability is \( 0.9983 \).

b) The area on the left is 0.0630.
Therefore the prob. is \( 0.9370 \) \( (1 - 0.0630) \).

c) The prob. is \( 0.9808 - 0.1423 = 0.8385 \).

d) \( \mu = 0 \), \( \sigma = \frac{\sigma}{\sqrt{n}} = \frac{1}{4} \).

\[ Z = \frac{0.27 - 0}{\left(\frac{1}{4}\right)} = 1.08 \]

The area on the left is 0.8599, so the prob. is \( 1 - 0.8599 = 0.1401 \).

2)

\( \mu = 21.1 \), \( \sigma = 5.1 \), \( n = 80 \)

a) The distribution is normal.

b) The mean of the sample means is \( \mu = 21.1 \).

c) The standard deviation of the sample means is

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.1}{\sqrt{80}} = 0.5702 \]

\[ Z = \frac{23 - 21.1}{0.5702} \approx 3.33 \]

The area on the left is 0.9996.
So the probability is \( 1 - 0.9996 = 0.0004 \).
3) \[ \mu = 163.4 \text{ mm}, \quad \sigma = 66 \text{ mm} \]

a) \[ z = \frac{1500 - 163.4}{66} = -13 \frac{1}{66} \approx -2.03 \]

The area on the left is 0.0212. Therefore, the probability is 1 - 0.0212 = 0.9788.

b) We want to find a z-score \( z \) such that the area to the left is 0.9500. The z-score is 1.645.

\[ z = 1.645 \]

It asks for the height so we translate it to a height:

\[ 1.645 = z = \frac{x - \mu}{\sigma} = \frac{x - 163.4}{66} \]

So:

\[ x = (1.645)(66) + 163.4 = 1742.57 \]

The answer is 1742.57 mm (you could round it).

Note: The question is ill-posed because the 95th percentile is applicable for the tallest 5% not for the lowest 95%.

4) a) Since the male has a carry-on you can assume \( x = 195 - 20 = 175 \).

\[ z = \frac{175 - 182.9}{40.9} = -0.19315 \approx -0.19 \]

So the area on the left is 0.4247.

So, the probability is 1 - 0.4247 = 0.5753.

b) Because of the carry on we want the sample mean to be greater than 175 instead of 195.

\[ \mu = 182.9 \quad \text{so} \quad \sigma_x = \frac{40.9}{\sqrt{13}} = 2.8024 \]
Therefore $z \approx \frac{175 - 182.9}{2.8024} = -2.82$.

The area to the left of $-2.82$ is 0.0024.

So the probability is $0.9976$

The probability is very high, so the pilot should be concerned.

(5) a) \[ \hat{p} = \frac{284}{557} = 0.50987 \approx 0.51 \]

b) 95% confidence interval.

Therefore $z_{0.025} = 1.96$

So $E = 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \sqrt{\frac{0.51 \times 0.49}{557}} = 0.0415$

$E \approx 0.042$

$\hat{p} - E < p < \hat{p} + E$

Therefore $0.468 < p < 0.552$

C) You cant since the confidence interval allows values below 0.5.

(6) We use the formula $n = \left(\frac{z_{0.025}}{E}\right)^2 (\frac{1}{4})$ because $\hat{p}$ is unknown.

Since the confidence level is 0.99, $z_{0.025} = 2.575$

So $n = \left(\frac{2.575}{0.02}\right)^2 \left(\frac{1}{4}\right) = 4144.14$

We round up to get $n = 4145$
7) a) Student $t$, because $\sigma$ is unknown and $n > 30$.

b) Normal, because $\sigma$ is known and $n > 30$.

c) None, because the population is skewed and to estimate the standard deviation we require it to be normal.

d) $X^2$, because we are estimating $\sigma$ and the population has a normal distribution.

e) Normal, because $n \geq 25$ ($850(0.1) = 85$) and $n' \geq 25$ ($850(0.9) = 765$).

8) a) $n = \left( \frac{z_{a/2}}{E} \right)^2 \left( \frac{1}{4} \right)$. Since $1 - \alpha = 0.98$ then $\alpha = 0.02$ so $z_{a/2} = 0.015$. Thus $1 - z_{a/2} = 0.99$.

Therefore $z_{a/2} = 2.33$ (using Table A.2).

So $n = \left( \frac{2.33}{0.05} \right)^2 \left( \frac{1}{4} \right) = 542.89$.

Therefore $n = 543$.

b) Since we're estimating the mean, $n = \left( \frac{z_{a/2} \sigma}{E} \right)^2$.

$\sigma = 337$, $E = 50$, $z_{a/2} = 2.33$.

So $n = \left( \frac{2.33 \times 337}{50} \right)^2 \left( \frac{1}{50} \right) = 246.62$.

$n = 247$
c) Since 543 is the largest of 543 and 247, we need a sample of at least 543.  
\[ n = 543 \]

9)

a) \[ \bar{x} = 42.79 \quad s = 5.69 \quad n = 7 \]

Since \( \sigma \) is unknown, we use the t-test.

1 - \( \alpha \) = .95 so \( \alpha = .05 \)

Therefore \[ t_{0.05} = 2.447 \] (using Table 4-3)

(note that there are 6 degrees of freedom and the area in two tails is .05)

\[ E = t_{0.05} \frac{s}{\sqrt{n}} = 2.447 \frac{5.69}{\sqrt{7}} = 5.179 \]

\[ \pm 5.18 \]

So the CI for the mean is

\[ \bar{x} - E < \mu < \bar{x} + E \]

\[ 37.52 < \mu < 47.88 \]

b) Since we were estimating \( \sigma \) we use the \( \chi^2 \) test.

1 - \( \alpha \) = .95 so \( \alpha = .05 \) so \[ \frac{\alpha}{2} = .025 \]

and \[ 1 - \frac{\alpha}{2} = .975 \]

For \( \chi^2 \) we use Table 4-4 with 6 degrees of freedom and area to the right = 0.025

so \[ \chi^2 = 14.449 \]

For \( \chi^2 \) we use Table 4-4 with 6 degrees of freedom and area to the right = 0.975

so \[ \chi^2 = 1.237 \]
The CI for $\sigma$ is

$$\left( \sqrt{\frac{n-1}{\chi^2}} \right) S \leq \sigma \leq \left( \sqrt{\frac{n-1}{\chi^2}} \right) S$$

$$\left( \frac{6}{\sqrt{14.449}} \right) (5.6) \leq \sigma \leq \left( \frac{6}{\sqrt{1.237}} \right) (5.6)$$

$$3.61 \leq \sigma \leq 12.33$$