

Homework 2 Solutions

Enrique Treviño

September 16, 2026

1 Section 3.1

Problem 1. (Exercise 1)

Four people are to be arranged in a row to have their picture taken. In how many ways can this be done?

Solution 1. $4! = 24$.

Problem 2. (Exercise 3)

In a digital computer, a *bit* is one of the integers $\{0, 1\}$, and a *word* is any string of 32 bits. How many different words are possible?

Solution 2. $2^{32} = 4294967296$.

Problem 3. (Exercise 7)

Five people get on an elevator that stops at five floors. Assuming that each has an equal probability of going to any one floor, find the probability that they all get off at different floors.

Solution 3. There are $5!$ ways of all of them choosing different floors and there are 5^5 total possible ways of choosing floors for each person, so the probability is

$$\frac{5!}{5^5} = \frac{120}{3125} = \frac{24}{625} = 0.0384.$$

Problem 4. (Exercise 10)

A deck of ordinary cards is shuffled and 13 cards are dealt. What is the probability that the last card dealt is an ace?

Solution 4. That it's the last card is irrelevant, the probability is

$$\frac{4}{52} = \frac{1}{13},$$

because there are 4 aces and 52 cards.

Problem 5. (Exercise 11)

There are n applicants for the director of computing. The applicants are interviewed independently by each member of the three-person search committee and ranked from 1 to n . A candidate will be hired if he or she is ranked first by at least two of the three interviewers. Find the probability that a candidate will be accepted if the members of the committee really have no ability at all to judge the candidates and just rank the candidates randomly. In particular, compare this probability for the case of three candidates and the case of ten candidates.

Solution 5. The probability that all three committee members choose different candidates as ranked first is

$$\frac{n(n-1)(n-2)}{n^3} = \frac{n^3 - 3n^2 + 2n}{n^3} = 1 - \frac{3}{n} + \frac{2}{n^2}.$$

Therefore, using the complement event, we get that the probability that someone will be accepted is

$$1 - \frac{n(n-1)(n-2)}{n^3} = \frac{3}{n} - \frac{2}{n^2}.$$

For $n = 3$, we get

$$1 - \frac{2}{9} = \frac{7}{9} = 0.777 \dots$$

For $n = 10$, we get

$$\frac{3}{10} - \frac{2}{100} = \frac{28}{100} = 0.28.$$

You can see it is much more likely with three candidates versus ten.

Problem 6. (Exercise 13)

A certain state has license plates showing three numbers and three letters. How many different license plates are possible

- (a) if the numbers must come before the letters?
- (b) if there is no restriction on where the letters and numbers appear?

Solution 6.

- (a) There are 26 letters in the alphabet and 10 possible digits. Therefore,

$$10^3 \cdot 26^3 = 17576000.$$

- (b) There are $\binom{6}{3} = 20$ ways of choosing where the numbers go, so the answer is

$$26^3 \cdot 10^3 \cdot \binom{6}{3} = 351520000.$$

2 Section 3.2

Problem 7. (Exercise 2)

In how many ways can we choose five people from a group of ten to form a committee?

Solution 7.

$$\binom{10}{5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5!} = 252.$$

Problem 8. (Exercise 3)

How many seven-element subsets are there in a set of nine elements?

Solution 8.

$$\binom{9}{7} = \binom{9}{2} = \frac{9 \cdot 8}{2} = 36.$$

Problem 9. (Exercise 8)

A die is rolled 30 times. What is the probability that a 6 turns up exactly 5 times? What is the most probable number of times that a 6 will turn up?

Solution 9. The probability that it turns up exactly 5 times is

$$\binom{30}{5} \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^{25} = \frac{786483287811279296875}{4093961476309876998144} = 0.1921 \dots$$

The most probable number of 6s is 5, because 6 should appear one out of every 6 on average. We calculate this exactly in Table 9, where for each value of k between 0 and 30, we write the probability that we rolled k 6's, which is

$$\binom{30}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{30-k}.$$

k	$P(X = k)$
0	0.00421272
1	0.0252763
2	0.0733013
3	0.136829
4	0.184719
5	0.192108
6	0.160090
7	0.109776
8	0.0631212
9	0.0308593
10	0.0129609
11	0.00471305
12	0.00149247
13	0.000413298
14	0.000100372
15	0.0000214128
16	4.01490×10^{-6}
17	6.61278×10^{-7}
18	9.55179×10^{-8}
19	1.20654×10^{-8}
20	1.32720×10^{-9}
21	1.26400×10^{-10}
22	1.03418×10^{-11}
23	7.19428×10^{-13}
24	4.19667×10^{-14}
25	2.01440×10^{-15}
26	7.74769×10^{-17}
27	2.29561×10^{-18}
28	4.91917×10^{-20}
29	6.78506×10^{-22}
30	4.52337×10^{-24}

Table 1: Probability distribution of the number of 6s after 30 dice rolls.

Problem 10. (Exercise 10)

In a ten-question true-false exam, find the probability that a student gets a grade of 70% or better by guessing. Answer the same question if the test has 30 questions, and if the test has 50 questions.

Solution 10. For ten questions, we have

$$\left(\frac{1}{2^{10}}\right) \left(\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right) = \frac{11}{64} = .171875.$$

For thirty questions, the probability is

$$\left(\frac{1}{2^{30}}\right) \left(\sum_{k=21}^{30} \binom{30}{k}\right) = \frac{22964087}{1073741824} = 0.021387.$$

For fifty questions, the probability is

$$\left(\frac{1}{2^{50}}\right) \left(\sum_{k=35}^{50} \binom{50}{k}\right) = \frac{928930468869}{281474976710656} = 0.00330022.$$

Problem 11. (Exercise 11)

A restaurant offers apple and blueberry pies and stocks an equal number of each kind of pie. Each day ten customers request pie. They choose, with equal probabilities, one of the two kinds of pie. How many pieces of each kind of pie should the owner provide so that the probability is about .95 that each customer gets the pie of his or her own choice?

Solution 11. Suppose the number of pies of each flavor is n . Clearly $n \geq 5$ because otherwise there aren't even enough pies. Also $n \leq 10$ because when $n \geq 10$, the probability is 1 (as only ten customers get a pie). Let's find the probability that a customer can't get a pie of their choice for each value of n between 5 and 10. Note that our goal is to get a probability under 0.05 (because it's the complement).

For $n = 5$, if 6 or more people wanted the same flavor, then someone doesn't get a pie of their choice, the probability of this outcome is

$$\frac{2}{2^{10}} \left(\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} \right) = \frac{772}{1024} > 0.05.$$

For $n = 6$, we get

$$\frac{2}{2^{10}} \left(\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} \right) = \frac{352}{1024} > 0.05.$$

For $n = 7$, we get

$$\frac{2}{2^{10}} \left(\binom{10}{8} + \binom{10}{9} + \binom{10}{10} \right) = \frac{112}{1024} > 0.05.$$

With $n = 8$, we get

$$\frac{2}{2^{10}} \left(\binom{10}{9} + \binom{10}{10} \right) = \frac{22}{1024} \leq 0.05.$$

Therefore, the owner needs to make 8 pies of each flavor. There is no way for the probability to be about 0.95%, with $n = 7$ the probability is about 89% and with $n = 8$ the probability is around 98%.

Problem 12. (Exercise 19)

A gin hand consists of 10 cards from a deck of 52 cards. Find the probability that a gin hand has

- (a) all 10 cards of the same suit.
- (b) exactly 4 cards in one suit and 3 in two other suits.
- (c) a 4, 3, 2, 1 distribution of suits.

Solution 12.

- (a) There are $\binom{4}{1}$ possible suits to choose from then $\binom{13}{10}$ ways of choosing the ten cards from the same suit, so we get

$$\frac{\binom{4}{1} \binom{13}{10}}{\binom{52}{10}} = \frac{1144}{15820024220} = 7.23 \times 10^{-8}.$$

- (b) There are $\binom{4}{1}$ ways of choosing the suit with 4 of the suit, then $\binom{13}{4}$ ways of choosing those 4 cards of the same suit. There are $\binom{3}{2}$ ways of choosing the two suits with 3 cards (because one suit was already picked, so only 3 suits left) and for each suit there are $\binom{13}{3}$ ways of choosing the 3 cards of that suit. Therefore, the probability is

$$\frac{\binom{4}{1} \binom{13}{4} \binom{3}{2} \binom{13}{3} \binom{13}{3}}{\binom{52}{10}} = \frac{701809680}{15820024220} = 0.0443621.$$

- (c)

$$\frac{\binom{4}{1} \binom{13}{4} \binom{3}{1} \binom{13}{3} \binom{2}{1} \binom{13}{2} \binom{1}{1} \binom{13}{1}}{\binom{52}{10}} = \frac{4976468640}{15820024220} = 0.314568.$$