Homework 7
Math 311: Introduction to Real Analysis

November 18, 2017

Problem 1. (**Exercise 4.3.1 (a, c, e)**) Determine the domain of convergence of the power series given below:

(a) $\sum_{n=1}^{\infty} n^3 x^n$.

(c) $\sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n$.

(e) $\sum_{n=1}^{\infty} \left(\frac{2+(-1)^n}{5+(-1)^{n+1}} \right)^n x^n$.

Problem 2. (**Exercise 4.3.4 (a,c)**) Suppose that the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$ is R, $0 < R < \infty$. Evaluate the radius of convergence of the following series:

(a) $\sum_{n=0}^{\infty} 2^n a_n x^n$.

(c) $\sum_{n=0}^{\infty} \frac{n^n}{n!} a_n x^n$.

Problem 3. (**Exercise 4.3.9**) Find the radius of convergence for

$$\sum_{k=1}^{\infty} \frac{2^k}{\sqrt{k}} x^k.$$

Problem 4. (**Exercise 4.4.9**) Prove that the following series converges:

$$\sum_{k=2}^{\infty} \frac{\sin\left(\frac{k}{100}\right)}{\ln k}.$$

Problem 5. (**Exercise 5.1.2**) Evaluate

$$1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} + \frac{1}{128} - \frac{1}{256} + \cdots .$$

Problem 6. (**Exercise 5.2.4**) Consider the power series expansion for sine:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^{2k-1}}{(2k-1)!}.$$

(a) Show that this series converges uniformly over the interval $[-\pi, \pi]$.

(b) How many terms must you take if the partial sum is to lie within $\epsilon = 1/2$ (for all $x \in [-\pi, \pi]$)?

(c) What about for $\epsilon = 1/10$.

Problem 7. Show using the definition of uniform continuity that $f(x) = \frac{x}{x+1}$ is uniformly continuous on the interval $[0, 2]$.

Problem 8. Prove that $f(x) = \sqrt{x}$ is uniformly continuous on the interval $[0, 1]$.