Homework 3 Solutions
Math 150

Enrique Treviño

3.1 True of false.

(a) False. If it is a fair coin, it will be 50% likely to land Heads the next time.

(b) False. There are face cards that are red.

(c) True. No face card is also an Ace.

3.3 Four games, one winner.

(a) 10 times. The probability of landing more than 60% of heads diminishes with more tosses, so you want the fewest tosses possible.

For those curious about the difference in probability: the probability of getting the proportion of heads larger than 0.60 on 10 tosses is

$$\frac{11}{64} \approx 17.2\%,$$

while the probability of getting the proportion of heads larger than 0.60 on 100 tosses is

$$\frac{139442359169148003731696 2055}{792281625142643375935439503 36} \approx 1.8\%.$$

(b) 100 times. With more tosses, the likelihood of ending around 50% is higher, so you want more tosses to make it more likely you win.

For those curious about the difference in probability: the probability of getting the proportion of heads larger than 0.40 on 10 tosses is

$$\frac{319}{512} \approx 62.3\%,$$

while the probability of getting the proportion of heads larger than 0.40 on 100 tosses is

$$\frac{1539491985769203636747955 55347}{1584563250285286751870879 00672} \approx 97.2\%.$$

(c) 100 times. With more tosses, the likelihood of ending around 50% is higher, so you want more tosses to make it more likely you win.

For those curious about the difference in probability: the probability of getting the proportion of heads between 0.40 and 0.60 on 10 tosses is

$$\frac{63}{256} \approx 24.6\%,$$

while the probability of getting the proportion of heads between 0.40 and 0.60 on 100 tosses is

$$\frac{747210360626560260812516 05011}{792281625142643375935439 50336} \approx 94.3\%.$$

(d) 10 times. The probability of landing less than 30% of heads diminishes with more tosses, so you want the fewest tosses possible.

For those curious about the difference in probability: the probability of getting the proportion of heads smaller than 0.30 on 10 tosses is

$$\frac{7}{128} \approx 5.5\%,$$

while the probability of getting the proportion of heads smaller than 0.30 on 100 tosses is

$$\frac{25479789183062789761893 25}{1584563250285286751870879006 72} \approx 0\%.$$

3.6 Dice rolls.

(a) 0. Two dice never add up to 1.

(b) $\frac{4}{36} = \frac{1}{9} \approx 11.1\%$.

(c) $\frac{1}{36} \approx 2.8\%$.

3.8 Poverty and language.

(a) No because 4.2% of people fall into both categories.

(b) The Venn diagram is:

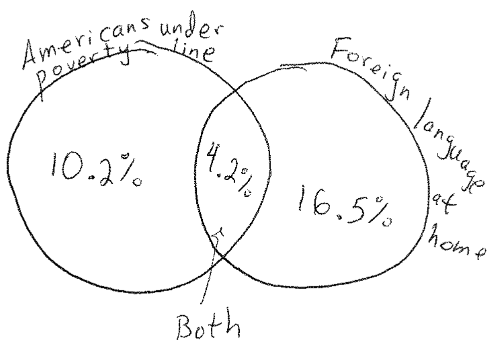

Figure 1: 10.2% should be 10.4%

(c) 10.4%

(d) $10.4\% + 4.2\% + 16.5\% = 31.1\%$.

(e) $100\% - 31.1\% = 68.9\%$.

(f) No, because $.207 \times .146 \neq .042$. It doesn't follow the multiplicative property for independent events.

3.10 Guessing on an exam.

(a)

$$\left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{1}{4}\right) = \frac{81}{1024} \approx 7.9\%.$$

(b)

$$\left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) \times \left(\frac{1}{4}\right) = \frac{1}{1024} \approx 0.1\%.$$

(c)

$$1 - \left(\frac{3}{4}\right)^5 = \frac{781}{1024} \approx 76.3\%.$$

3.12 School absences.

(a) $1 - .25 - .15 - .28 = .32$. 32%.

(b) $1 - .15 - .28 = .57$. 57%.

(c) $1 - .32 = .68$. 68%.

(d) Assuming that each kid misses school independently, $.32 \times .32 = 0.1024 = 10.24\%$.

(e) Under the same assumption as above $.68 * .68 \approx 0.4624 = 46.24\%$.

(f) It's not a reasonable assumption. If the kids are in the same household, they don't get sick independently. Furthermore, if one kid gets sick, the other might fake sickness or the parents might decide to let the other skip class to join the sibling.

3.13 Joint and conditional probabilities.

(a) No. We could if we knew that A and B are independent events.

(b)

i. $(0.3)(0.7) = (0.21)$.

ii. $0.3 + 0.7 - 0.21 = 0.79$.

iii. 0.3.

(c) No, because if they were independent $P(A \text{ and } B) = 0.21$, but $0.1 \neq 0.21$.

(d)

$$\frac{0.1}{0.7} = \frac{1}{7} \approx 0.143.$$

3.14 PB & J. Let A be the event that a person likes peanut butter and B that a person likes jelly. Then we know that $P(A) = 0.80$, $P(B) = 0.89$, and $P(A \text{ and } B) = 0.78$. We want to find $P(B|A)$, therefore

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.78}{0.80} = 0.975.$$

Therefore 97.5%.

3.16 Health coverage, relative frequencies.

(a) No, because there are people that have both.

(b) 0.2329.

(c) $\frac{0.2099}{0.8738} \approx 0.18$.

(d) $\frac{0.0230}{0.1262} \approx 0.24$.

(e) No because the answers to (c) and (d) are not the same as the answer to (b) (which is what would happen if they were independent.

3.23 Marbles in an urn.

(a) $\frac{3}{10} = 0.3.$

(b) $\frac{3}{10} = 0.3.$

(c) $\frac{3}{10} = 0.3.$

(d) $\left(\frac{3}{10}\right)\left(\frac{3}{10}\right) = 0.09.$

(e) Yes because we replace the balls so every draw is just like the first draw.

3.24 Socks in a drawer.

(a)

$$\frac{4 \times 3}{12 \times 11} = \frac{1}{11} \approx 0.091.$$

(b)

$$\frac{7 \times 6}{12 \times 11} = \frac{7}{22} \approx 0.318.$$

(c)

$$\frac{3 \times 2 + 3 \times 9 + 9 \times 3}{12 \times 11} = \frac{60}{132} = \frac{15}{33} \approx 0.455.$$

(d) 0. There are no green socks.

(e)

$$\frac{3 \times 2 + 4 \times 3 + 5 \times 4}{12 \times 11} = \frac{38}{132} = \frac{19}{66} \approx 0.288.$$

3.26 Books on a bookshelf.

(a)

$$\frac{28 \times 59}{95 \times 94} = \frac{1652}{8930} \approx 0.185.$$

(b)

$$\frac{72 \times 27}{95 \times 94} = \frac{1944}{8930} \approx 0.218.$$

(c)

$$\left(\frac{72}{95}\right)\left(\frac{28}{95}\right) = \frac{2016}{9025} \approx 0.223.$$

(d) That is because the proportion of hardcover books changed from $\frac{28}{95}$ to $\frac{27}{94}$, the difference is very small (only .008) and then this difference is multiplied by 72/95, making the difference even smaller (.005). The difference is less than half of a percent!

3.40 Health coverage, frequencies.

(a)

$$\frac{459}{20000} \approx 0.023 = 2.3\%.$$

(b)

$$\frac{4657 + 2524 - 459}{20000} = \frac{6722}{20000} = 0.3361 = 33.61\%.$$

3.41 HIV in Swaziland. Let's make a table summarizing what we are given:

		ELISA Result		
		HIV +	HIV -	Total
Has	No	0.0548	0.6862	0.741
HIV	Yes	0.2582	0.0008	0.259
	Total	0.3130	0.6870	1.0000

The table was calculated in the following way: The percentage of people that ELISA labels HIV + and that have HIV are $0.259 \times 0.997 \approx 0.2582$ because the test is 99.7% accurate. The number 0.0008 comes from subtracting 0.2582 from 0.259. The percentage of people that ELISA labels HIV - and that don't have HIV are $0.741 \times 0.926 \approx 0.6862$ because ELISA is 92.6% accurate. Then finally to get the last entry, we subtract 0.7047 from 0.741. The last row is formed by adding the columns.

To get the desired probability, we want the probability of having HIV when the test comes out positive. So the answer is

$$\frac{0.2582}{0.3130} \approx 0.8247.$$

Therefore, the probability is about 82.5% which is considerably lower than 99.7% (which is what one would naively think).

3.42 Twins. Let A be the event with both girls and B be the event that the twins are identical. We want to find the probability that they are identical, given that they are both girls, therefore we want to find $P(B|A)$. We know $P(B) = 0.30$. Let's calculate $P(A)$:

$$P(A) = 0.30 \times \left(\frac{1}{2}\right) + 0.70 \times \left(\frac{1}{4}\right) = 0.15 + 0.175 = 0.325.$$

The last piece of the puzzle to be able to find $P(B|A)$ is to find $P(B \text{ and } A)$.

$$P(B \text{ and } A) = 0.30 \times \left(\frac{1}{2}\right) = 0.15.$$

Therefore

$$P(B|A) = \frac{0.15}{0.325} \approx 0.4615.$$

Therefore the probability that the twins are identical is approximately 46.2% (note, it is a slight increase over 30% if we had no information that they were both girls).