Homework 7 Solutions
Math 150

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7.24 Diamonds, Part I. The hypothesis are:

Null Hypothesis: The average standardized prices of 0.99 carat diamonds is the same as the average standardized prices for 1 carat diamonds.

Alternative Hypothesis: The average standardized prices of 0.99 carat diamonds is less than the average standardized prices for 1 carat diamonds.

In symbols (letting μ be the average of 1 carat diamond standardized prices minus the average standardized prices of the 0.99 carat diamonds):

$\mathbf{H_0}$: $\mu = 0$.

$\mathbf{H_A}$: $\mu > 0$.

The observations are independent since they come from simple random samples of less than 10% of the population (there are many more than 230 diamonds, so 23 is less than 10%). The boxplots suggest that the distributions are a skewed. But they are not very skewed, so giving the sample size of 23, it is reasonable to assume near-normality.

$$SE = \sqrt{\frac{13.32^2}{23} + \frac{16.13^2}{23}} \approx \sqrt{\frac{437.5993}{23}} \approx \sqrt{19.026} \approx 4.3619.$$

Therefore the t-score is

$$t = \frac{56.81 - 44.51}{4.3619} = \frac{12.3}{4.3619} \approx 2.82.$$

We have df $= 22$ (we are erring on the safe side by taking one less than the minimum of the sample sizes). Using table $B.2$ we have that the p-value for the one-sided test is approximately 0.005. We have enough evidence to reject the null hypothesis in favor of the alternative hypothesis. That is, we have evidence that suggests that the price of 1 carat diamonds is inflated disproportionally to the number of carats.

7.26 Diamonds, Part II. We are erring on the safe side and using df $= 22$. Since it's a 95% confidence interval, then $t^* = 2.07$ (using table $B.2$). We also know from work on the previous exercise that $SE \approx 4.3619$. Therefore the confidence interval is

$$12.3 \pm 2.07 \times 4.3619 \approx 12.3 \pm 9.03.$$

Therefore the confidence interval is $(3.27, 21.33)$.

7.28 Fuel efficiency of manual and automatic cars, Part I.

$$SE = \sqrt{\frac{3.58^2}{26} + \frac{4.51^2}{26}} = \sqrt{\frac{33.1565}{26}} = \sqrt{1.27525} \approx 1.1293.$$

Therefore

$$t = \frac{19.85 - 16.12}{1.1293} = \frac{3.73}{1.1293} \approx 3.30.$$

Using df $= 25$ and that $3.30 > 2.79$, we can see that the p-value is smaller than 0.01. Therefore we have strong evidence to reject the null hypothesis in favor of the alternative hypothesis. That is, there is evidence that suggests that manual transmission cars are more fuel efficient than automatic cars.

Note: This study would have been better if we had paired data. Comparing the same car with a manual transmission versus an automatic transmission.

7.47 Gaming and distracted eating, Part I.

$$SE = \sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}} = \sqrt{\frac{2730.97}{22}} = \sqrt{124.135} \approx 11.1416.$$

Therefore

$$t = \frac{52.1 - 27.1}{11.1416} = \frac{25}{11.1416} \approx 2.24.$$

Using df $= 21$, using table $B.2$ we see that because $2.07 < 2.24 < 2.50$, then the p-value is between .02 and .05 (it is a two sided test, so we use the numbers from two tails). Therefore we have enough evidence to reject the null hypothesis in favor of the alternative hypothesis, i.e., there is strong evidence that the average food intake is larger for people eating while playing solitaire.

7.56 Age at first marriage, Part I.

$$SE = 1.96 \times \frac{4.72}{\sqrt{5534}} = 1.96 \times 0.063 \ldots \approx 0.1244.$$

Therefore the 95% confidence interval is

$$23.44 \pm 0.1244 = (23.2156, 23.5644).$$

That means we're 95% confident that the average age of woman at first marriage is between 23.22 and 23.56.

The sample size is so large that whether the distribution is skewed or not is irrelevant. We're also assuming that the data comes from a random sample of less than 10% of the population. Since 5536 is much smaller than 10% of the population of American women who have married, we can use the Central Limit Theorem to create the confidence interval.

7.58 Age at first marriage, Part II. The null hypothesis and the alternative hypothesis are statements about the parameter μ not about the sample mean. So $\bar{x}$ should not be in the hypotheses. Another error is that the researcher is interested to find if there's an increase or a decrease, so the alternative hypothesis should be 2-sided.

For those curious, the correct hypotheses are:

$$H_0 : \mu = 23.44 \; years \; old.$$

$$H_A : \mu \neq 23.44 \; years \; old.$$