Homework 6 Solutions
Math 150

Enrique Treviño

7.1 Identify the critical t.

(a) df = 5, and $t^* = 2.02$. To find t^*, we look at table $B.2$.

(b) df = 20, and $t^* = 2.53$.

(c) df = 28, and $t^* = 2.05$.

(d) df = 11, and $t^* = 3.11$.

7.2 t-distribution. The solid line is the normal distribution. The dashed line is the t-distribution with 5 degrees of freedom. The dotted line is the t-distribution with 1 degree of freedom. The reason is that the less degrees of freedom, the 'thicker' the tails. Clearly the thickets tail is for the dotted line, followed by the dashed line and the solid line (in that order).

7.4 Find the p-value, Part II.

(a) df = 25. Using the table with row 25 we see that 2.485 is between 2.06 and 2.49, so the two sided tail has area between 0.020 and 0.05. It's much closer to 0.020 since $2.485 \approx 2.49$, but it is still larger than 0.010, therefore the null hypothesis is not rejected at $\alpha = 0.01$.

(b) df = 17. $0.5 < 1.33$, so the p-value is larger than 0.200. Therefore the null hypothesis is not rejected at $\alpha = 0.01$.

7.8 Heights of adults.

(a) The point estimate for the average is 171.1. For the median it is 170.3.

(b) For standard deviation: 9.4. For IQR: $177.8 - 163.8 = 14$.

(c) The 180cm person is not unusually tall because, the person is rougly one standard deviation from the mean. Not unusual. The 155cm is roughly 1.63 standard deviations away from the mean, so it's also not unusually short.

(d) No. I expect them to be close, but not exactly the same. There is some variability for different samples.

(e) The measure is the standard error and it comes out to:

$$SE = \frac{s}{\sqrt{n}} = \frac{9.4}{\sqrt{507}} \approx 0.417.$$

7.12 Auto exhaust and lead exposure.

(a) In words, the hypothesis are:

Null Hypothesis: The average lead concentration for police officers is the same as the average for people not exposed to automobile exhaust fumes, i.e., $mu = 35$.

Alternative Hypothesis: The average lead concentration for police officers is greater than the average for people not exposed to automobil exhaust fumes, i.e., $\mu > 35$.

In symbols:

$\mathbf{H_0}$: $\mu = 35$.

$\mathbf{H_A}$: $\mu > 35$.

(b) We are not told whether the police officers are randomly sampled. It's unclear whether the independence condition is satisfies. The researchers are interested in lead exposure in general, so testing police officers is a convenience sample, not a random sample. The near-normality condition is hard to check without more data, but the sample size is 52, so it's probably big enough to hold.

(c) Under the assumption that the police officers were randomly sampled, we can test under the context of whether police officers in urban settings have higher lead concentration than people living in nearby suburbs. In other words, the study cannot be generalized to the population as a whole. With that being said, let's test it.

$$t = \frac{124.32 - 35}{\left(\frac{37.74}{\sqrt{52}}\right)} \approx \frac{89.32}{5.2336} \approx 17.07.$$

The t-score is enormous, so the p-value is minuscule. Therefore there is enough evidence to reject the null hypothesis in favor of the alternative hypothesis.

(d) No. The p-value is very very very small. Much smaller than .005 (which is the size it would have to be to be included in the 99% confidence interval).

7.14 SAT scores.

(a) The margin of error is $t^* \times SE$. For a 90% confidence interval, we have that t^* is the value in the second column of $B.2$. It depends on the degrees of freedom which would be $n - 1$. We know that $t^* \geq 1.28$. We also know $SE = 250/\sqrt{n}$. So we have

$$\begin{aligned} \frac{250t^*}{\sqrt{n}} &= 25 \\ \frac{250}{25}t^* &= \sqrt{n} \\ 100(t^*)^2 &= n. \end{aligned}$$

Therefore

$$n \geq 100 \times (1.28)^2 \approx 163.84.$$

If df $= 150$ or df $= 200$, then $t^* = 1.29$, therefore we need

$$n \geq 100 \times (1.29)^2 \approx 166.41.$$

Therefore if $n \geq 167$, then we can get a margin of 25 for a 90% confidence interval.

(b) It has to be larger, because t^* would be bigger for a 99% confidence interval.

(c) The idea is similar, but since it is a 99% confidence interval we have $t^* \geq 2.58$. That means that

$$n \geq 100 \times (2.58)^2 \approx 665.64.$$

Given that the table only gets to 500, we'll have to settle for using $t^* \approx 2.59$ for the large n, so we get

$$n \geq 100 \times (2.59)^2 \approx 670.81.$$

Therefore we need at least 671 to be sampled.

7.15 Air quality.

(a) Two sided test because we want to compare the years. We don't have a reason to suspect it will be higher or lower.

(b) We should use a paired test because we can compare the same cities to themselves.

(c) t-test because the sample is small and the population standard deviation is unknown.

7.16 True/False: paired.

(a) True. (Note: The sentence is ambiguous, if one interprets "each pair" as every possible pair of observations from the data set, then it the statement would be false. Given the context it is more reasonable to interpret "each pair" as the pairs from the paired data).

(b) True. For a matched pair analysis, we need to pair up each observation from one data set with an observation from the other data set. They have to have the same sample sizes.

(c) True.

(d) False. We subtract each pair of "paired" observations. (Note: If the inference is done with this technique, then the data has the same point estimate for the mean of the differences, but it has a different standard deviation).

7.18 Paired or not, Part II?

(a) Paired.

(b) Paired. The natural correspondence is an item in one store versus the same item in the other store.

(c) Not paired. There is not a natural correspondence from an individual student in one school to an individual student in the other.

7.20 High School and Beyond, Part I.

(a) The relevant data is the histogram, since it's the distribution of the paired differences. There seems to be more negative differences than positive differences. The proportion is slight, without knowing the actual numbers, one can't conclusively say it's clear. (Note: If one looks at the box plots, the box plots don't have clear evidence one way or another because the medians and quartiles are about the same. However, looking at the box plots is not relevant for this question.)

(b) No.

(c) In words, the hypothesis are:

Null Hypothesis: The average scores in the reading exams and in the writing exams are the same, i.e., $\mu_{\text{read}} = \mu_{\text{write}}$. Therefore $\mu_{\text{diff}} = \mu_{\text{read}} - \mu_{\text{write}} = 0$.

Alternative Hypothesis: The average scores in the reading exams and in the writing exams are different, i.e., $\mu_{\text{read}} \neq \mu_{\text{write}}$. Therefore $\mu_{\text{diff}} = \mu_{\text{read}} - \mu_{\text{write}} \neq 0$.

In symbols:

$\mathbf{H_0}$: $\mu_{\text{diff}} = 0$.

$\mathbf{H_A}$: $\mu_{\text{diff}} \neq 0$.

(d) We have a simple random sample (it's from a survey, so it might not be random enough) and the sample is of less than 10% of the population. So we may assume the observations are independent. The distribution of the differences is nearly normal. It's a little skewed, but with $n = 200$, the skewness does not impact the results.

(e)

$$t = \frac{-0.545 - 0}{\left(\frac{8.887}{\sqrt{200}}\right)} \approx \frac{-0.545}{0.6284} \approx -.867.$$

Given that it's a two-sided test, the p-value with df = 199 for that t-score is bigger than 0.2, therefore there is not evidence to reject the null hypothesis.

(f) Since we didn't reject the null hypothesis, the possible error is a Type 2 error (failing to reject the null when the null is false). In the context, a Type 2 error means that there is a difference in the scores of students in the reading and writing sections but that we didn't detect it.

(g) Yes, because the null hypothesis is not rejected, so 0 is a plausible value.

7.22 High school and beyond, Part II.

(a) The sample size is 200, so df = 199. The table includes values for df = 150 and df = 200. For a 95% confidence interval, the row of df = 150 suggests using $t^* = 1.98$, for df = 200, it suggests $t^* = 1.97$. We'll err on the side of caution and use 1.98 (but using 1.97 is also acceptable):

$$\bar{x} \pm t^* \times SE = -0.545 \pm (1.98) \times \left(\frac{8.887}{\sqrt{200}}\right) \approx -0.545 \pm 1.244.$$

Therefore an approximation of the confidence interval is $(-1.789, 0.699)$.

Note: If you use $t^* = 1.97$, then the confidence interval comes out to $(-1.783, 0.693)$. Using statistical software, it calculates the confidence interval as $(-1.784, 0.694)$.

(b) We are 95% confident that the average difference between the reading portion of the test and the writing portion of the test is between -1.789 and 0.699.

(c) No, because 0 is a plausible average difference